KINGDOM OF BAHRAIN

Ministry of Education

مَمْلَكَةُ البَحْرَيْن

وَزَارَةُ التَّرْبِيَةِ وَالتَّعْلِيم

FIN 316

Financial Mathematics (2)

Secondary Level

2030 البحرين BAHRAIN

The Ministry of Education, Kingdom of Bahrain has decided to teach this book in secondary schools

Financial Mathematics (2)

316 مال

For Secondary Education

First Edition

2024 - 1445

Authoring and Development
A specialized team from the Ministry of Education
and specialists from the Kingdom of Bahrain

H.M. SHAIKH HAMAD BIN ISA AL KHALIFA

THE KING OF THE KINGDOM OF BAHRAIN

Content

Introduction

Financial mathematics is a branch of mathematics that focuses on analyzing data and modeling financial market. There are many potential applications in financial mathematics that includes:

- Economics, statistics and probability.
- Learning financial mathematics often includes understanding financial formulas and other math's skills.

This course will help the students to acquire many skills that will qualify them to join the universities education as well as the labor markets requirements.

By studying this course, our students will be able to know the method of calculating interest in banks and all methods of consuming loans using equal payments. The students will also be able to evaluate the new projects through different methods such as the payback period, net present value and internal rate of return. Other skills that will be acquired through this course.

General Objective

At this course, our students will learn:

1. calculate compound interest.
2. computing amount of annuities and loans amortization.
3. evaluating the project and capital budgeting decision model.
4. knowing and calculate breakeven point.
5. calculating ratio analysis.

The content of this book divided into five units address as follow:

Unit One : Compound Interest

Unit Two : Annuities and Loans Amortization

Unit Three : Capital Budgeting Decision Model

Unit Four : Breakeven Analysis

Unit Five : Ratio Analysis

Book Aid

Graphic illustrations, mind maps, tables and diagrams helps the students to assist their learning. Meaning of certain concepts have also been highlighted using specific symbols called icons. The purpose of these icons is to emphasize, draw their attention to important aspects of the work and to highlight of the activities. Various icons have the following meanings:

1	**Definition**	Help to identify and understand important concepts.
2	**Important Points**	Key concepts that need to be understood.
3	**Tips**	Making your work easier.
4	**Class Activity**	Benefit the students from opportunity to reflect upon their learning, align to outcomes and assessments.
5	**Reading**	Improve, empathy, memory, focus and communication skills.
6	**Internet Searching**	Quickly find what we looking without having to sift through pages of irrelevant results.
7	**Examples**	Practical questions solved to help understand the topic of the lesson.

Unit 1

Compound Interest

Learning Objective

At this unit, our students will learn:

- The difference between SI & CI (using simple interest equation).
- The calculation of compound interest.
- The calculation of future and present value.
- The difference between nominal and partial interest rates.
- The calculation of CI in more than once a year situation.
- The calculation of compound interest for changeable interest rate.
- The calculation of compound interest for changeable investment/loan.
- The calculation of the factors of compound interest.

1.1 Simple and Compound Interest

Meaning of Compound and Simple Interest

Differentiating between Compound and Simple Interest:

Interest is the cost of borrowing money, where the borrower pays a fee to the lender for the loan. The interest, typically expressed as a percentage, can be either simple or compounded. Simple interest is based on the principal amount of a loan or deposit. In contrast, compound interest is based on the principal amount and the interest that accumulates on it in every period. Simple interest is calculated only on the principal amount of a loan or deposit, so it is easier to determine than compound interest.

Simple Interest is the cost of borrowing. It is the interest only on the principal amount as a percentage of the principal amount. Borrowers will benefit simple interest by paying interest only when loans approved. In other words, it is the paid interest only loans approved. **Simple Interest** is the amount that one pays to the borrower for using the borrowed money for a fixed period.

One can easily compute simple interest by multiplying the interest amount with the tenure and the principal amount.

Compound Interest is different from simple interest, which gains interest only on principal sum, earns interest on the previously earned interest. The interest is added to the principal amount. Compound Interest is simply interest above interest. The whole principle revolves around generating high returns by compounding the interest received on the principal sum.

In other words, Compound interest has the potential to earn more return than just the simple interest from an investment. The investment grows exponentially with compound interest because it is based on the principal power of compounding. The most important concepts in finance and investment is compound interest. When an investment begins to pay interest on interest then compounding of interest takes place, and this is

termed "compound interest". This concept has been extended to "rates of return" which are said to be "compounded" over a time with multiple intervals.

Interest is what you're charged when you borrow money. Most commonly, you are assessed interest charges when taking out a loan from a bank or making purchases on a credit card. It is often expressed as APR, which stands for annual percentage rate. When taking out a loan, two main types of interest can be applied: simple interest or compound interest (see figure 1-1-1). Interest can be compounded at any interval, and the most common compounding intervals are:

- Annually: 1 time per year.
- Semiannually: 2 times per year.
- Thirdly: 3 times per year.
- Quarterly: 4 times per year
- Monthly: 12 times per year.
- Weekly: 52 times per year.
- Daily: 365 times per year.

Figure (1-1-1): Type of Interest

Defining Compound Interest:

Compound interest is the most important concepts in finance and investment. When an investment begins to pay interest on interest then compounding of interest takes place, and this is termed "Compound Interest". This concept has been extended to "Rates of Return" which are said to be "compounded" over a time period with multiple intervals.

Compound Interest:

- Compounded interest is interest calculated on the amount that includes the principal plus accumulated interest of the previous period.

When you borrow money from a bank, you will pay interest. Interest is basically a fee charged for borrowing the money, it is a percentage charged on the principal amount for a period of time usually a year.

If you want to know how much interest you will earn on your investment or if you want to know how much you will pay above the cost of the principal amount on a loan or mortgage, you will need to understand how compound interest works.

Interest is computed on the principal sum plus the accrued interest. At the beginning of the new interest period, all interest is added to the principal, forming a new principal figure on which interest is then calculated. This process repeats itself as each interest period-interest may be compounded daily, monthly, semiannually or annually.

Tips 1-1-1:

- The ability to calculate the value of money at different points in time is one of the most important skills you will develop in finance. It is the key to understanding the material ahead as well as making financial choices for your future. The time value of money (TVM) refers to a dinar in hand today being worth than a dinar received in the future. Today's dinar can be invested in an interest account that grows the value over time.

Simple Interest vs Compound Interest:

The major difference between simple interest and compound interest is that simple interest is based on principal amount whereas compound interest is based on the principal amount and the interest compounded for a cycle of the period.

Tips 1-1-2:

- The compound interest and simple interest are equal in the first year, and the difference between them starts from the second year (see figure 1-1-2), where the interest is added to the principal in the compound interest, while the principal of simple interest is fixed and does not change.

$$SI = PV \times I \times T$$

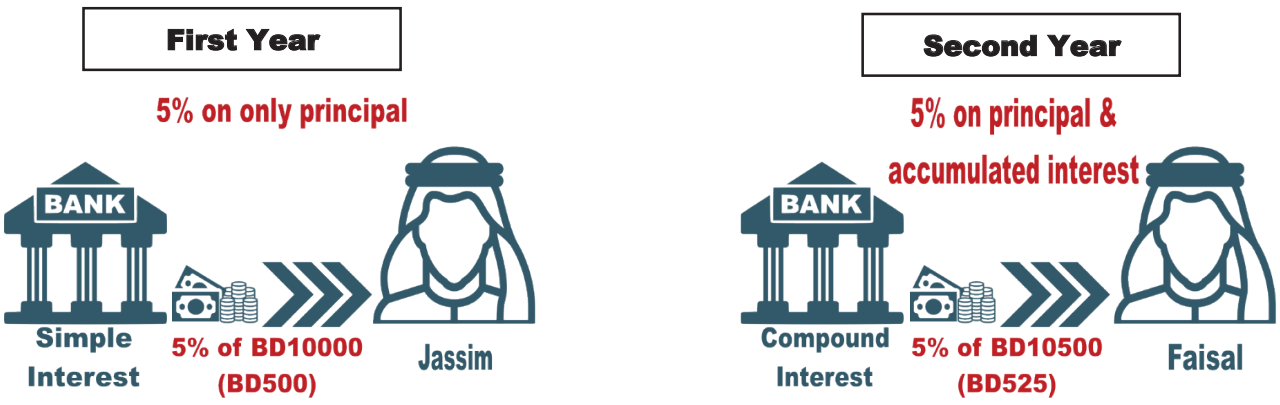


Figure (1-1-2): Simple vs Compound Interest

We know that simple interest and the compound interest are the two important concepts widely used in many financial services most especially in banking purposes. Loans such as instalments loans, car loans, educational loans, mortgages use simple interest. The compound interest is used by most of the savings' account as it pays the interest. It pays more than the simple interest. In the below example, let us discuss the difference between the simple interest and the compound interest in detail.

Example 1-1-1:

► A Person invested BD1000 in a bank for two years at 7% annually. Find the simple interest and compound interest at the end of each year.

Answer:

Simple Interest	Year	Compound Interest
SI = PV × I × T SI = 1000 × 7% × 1 = BD 70 A = PV + SI A = 1000 + 70 = BD 1070	First Year	SI = PV × I × T SI = 1000 × 7% × 1 = BD 70 A = PV + SI A = 1000 + 70 = BD 1070
SI = PV × I × T SI = 1000 × 7% × 1 = BD 70 A = PV + SI A = 1000 +70+70 = BD 1140	Second Year	SI = PV × I × T SI = 1070 × 7% × 1= BD 74.900 A = PV + SI A =1070 +74.900 = BD1144.900

Important Points 1-1-1:

- From the previous example, we can notice the following:

1. Compound interest = Simple interest at the first year.
2. Starting from the second year, the compound interest > simple interest.
3. The additional interest (BD70) earned up the first year to principal or original (BD1000) reflects the compounding of interest.

In the below table (1-1-1) you can find the key differences between Simple Interest and Compound Interest:

Simple Interest	Compound Interest
▶ Calculated from the principal at the end of the period.	▶ Calculated from the amount at the end of each period.
▶ Principal is fixed throughout the investment period.	▶ Principal is increasing throughout the investment period.
▶ Interest is fixed throughout the investment period	▶ Interest is increasing throughout the investment period.

Table (1-1-1): Simple vs Compound Interest

Activity 1-1-1:

- Find the difference between the simple interest and the compound interest if BD2000 is invested for two years at 4% annually at the end of each year.
- Ahmed deposited BD3500 in a bank at interest rate 6.25% annually for 3 years.

Required:

a) Find the amount at the end of 3 years (using simple interest rate).

b) Find the amount at the end of 3 years (using compound interest rate).

c) Find the difference between the amount of simple interest and the amount of compound interest.

1.2 Future Value and Compound Interest

Most users encounter the concept of compound interest at an early age. Anyone who has ever had a savings account or purchased a government savings bond has compound interest.

Compound Interest occurs when interest paid on the investment during the first period is added to the principal; then, during the second period, interest is earned on this new sum.

Future value (FV), which is the cash value of an asset (money, in the example) in the future that is equivalent in value to a specific amount today. In other words, single amount of money deposited today will grow into a larger amount tomorrow.

Another way to calculate the future value of the saving account is to multiply the deposit time's one and the interest rate for each year the money remains in the savings account.

Will you wait two years instead of one year for your lamp-sum payment? What is the future value of the deposit be after two years? This question will be answered through the following example.

Future Value = Present Vale x (1 + interest rate) $^{\text{Number of periods}}$

Calculate Future Value & Interest

The numerical value of $(1+i)^n$ can be computed by using an electronic calculator or you can use the facilities which are provided in the Microsoft office Excel to compute the compound amount for one monetary unit with a certain formula. You can easily appreciate the use of table No.1 when you consider how much time it saves. The calculation involves a lot of work and many chances to make mistakes if many periods are involved.

Important Points 1-2-1:

Future Value = Present Vale x (1 + interest rate) $^{\text{Number of periods}}$

$$FV = PV \times (1+ i)^{n}$$

Tips 1-2-1:

- Where, FV = the future value of the investment at the end of n years.
- PV = the present value, or original amount invested at the beginning of the first year.
- i = the annual interest (or discount) rate.
- n = the number of years during which the compounding occurs.

The factor $(1+i)^n$ is called the compound amount of one monetary unit (BD, $, € ……) that had compound interest rate of (i) and kept for (n) number of periods.

TABLE No.1 Equation: $FV = PV \times (1 + i)^n$ FV for 1BD

n \ i	0.5%	0.75%	1%	1.25%	1.5%	1.75%	2%	2.25%	2.5%
1	1.0050	1.0075	1.0100	1.0125	1.0150	1.0175	1.0200	1.0225	1.0250
2	1.0100	1.0151	1.0201	1.0252	1.0302	1.0353	1.0404	1.0455	1.0506
3	1.0151	1.0227	1.0303	1.0380	1.0457	1.0534	1.0612	1.0690	1.0769
4	1.0202	1.0303	1.0406	1.0509	1.0614	1.0719	1.0824	1.0931	1.1038
5	1.0253	1.0381	1.0510	1.0641	1.0773	1.0906	1.1041	1.1177	1.1314
6	1.0304	1.0459	1.0615	1.0774	1.0934	1.1097	1.1262	1.1428	1.1597
7	1.0355	1.0537	1.0721	1.0909	1.1098	1.1291	1.1487	1.1685	1.1887
8	1.0407	1.0616	1.0829	1.1045	1.1265	1.1489	1.1717	1.1948	1.2184
9	1.0459	1.0696	1.0937	1.1183	1.1434	1.1690	1.1951	1.2217	1.2489
10	1.0511	1.0776	1.1046	1.1323	1.1605	1.1894	1.2190	1.2492	1.2801

*Most standard calculators have a power function key (y^x) that allows us to raise $(1+i)^n$.

Calculate Compound Interest

The compound interest (CI) is calculated as the difference between future value and present value as follows:

Important Points 1-2-2:

Compound Interest = Future Value - Present Vale

$$CI = FV - PV$$

OR

Compound Interest = Present Vale x [(1 + interest rate) $^{\text{Number of periods}}$ - 1]

$$CI = PV \times [(1+ i)^n - 1]$$

Example 1-2-1:

► A trader borrowed BD3000 from a bank for 12 years at 8% annually. Find the future value and compound interest at the end of the period.

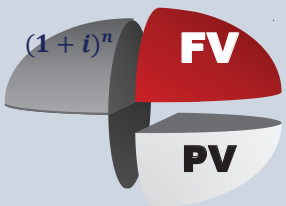
Answer:

1- Future Value:

$FV = PV \times (1+ i)^n$

$= 3000 \times (1+ 8\%)^{12}$

$= 3000 \times 2.5182 = \text{BD } 7554.600$



Using FV table for 1BD to find value of $(1+8\%)^{12}$

(by searching under I = 8% and n = 12 periods)

TABLE No.1 Equation: FV = PV × (1 +i)^n					FV for 1BD				
N	7.25%	7.5%	7.75%	8%	8.25%	8.5%	8.75%	9%	9.25%
1	1.0725	1.0750	1.0775	1.0800	1.0825	1.0850	1.0875	1.0900	1.0925
2	1.1503	1.1556	1.16101	1.1664	1.1718	1.1772	1.1827	1.1881	1.1936
3	1.2336	1.2423	1.25098	1.2597	1.2685	1.2773	1.2861	1.2950	1.3040
4	1.3231	1.3355	1.34794	1.3605	1.3731	1.3859	1.3987	1.4116	1.4246
5	1.4190	1.4356	1.4524	1.4693	1.4864	1.5037	1.5211	1.5386	1.5563
6	1.5219	1.5433	1.56496	1.5869	1.6090	1.6315	1.6542	1.6771	1.7003
7	1.6322	1.6590	1.68625	1.7138	1.7418	1.7701	1.7989	1.8280	1.8576
8	1.7506	1.7835	1.81693	1.8509	1.8855	1.9206	1.9563	1.9926	2.0294
9	1.8775	1.9172	1.95774	1.9990	2.0410	2.0839	2.1275	2.1719	2.2171
10	2.0136	2.0610	2.10947	2.1589	2.2094	2.2610	2.3136	2.3674	2.4222
11	2.1596	2.2156	2.27295	2.3316	2.3917	2.4532	2.5161	2.5804	2.6463
12	~~2.3162~~	~~2.3818~~	~~2.4491~~	2.5182	2.5890	2.6617	2.7362	2.8127	2.8911
3	2.4841	2.5604	2.63891	2.7196	2.8026	2.8879	2.9756	3.0658	3.1585

Example 1-2-1:

Answer:

2- **Compound Interest:**

$$CI = FV - PV$$
$$= 7554.600 - 3000$$
$$= BD\ 4554.600$$

Or

$$CI = PV \times [(1+i)^n - 1]$$
$$= 3000 \times [(1+8\%)^{12} - 1]$$
$$= 3000 \times (2.5182 - 1)$$
$$= 3000 \times 1.5182$$
$$= BD\ 4554.600$$

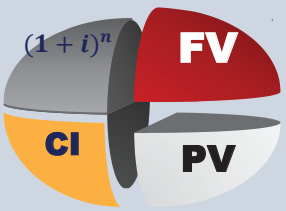


TABLE No.1	Equation : FV = PV × (1 + i)^n							FV for one monetary unit	
n \ i	7.25%	7.5%	7.75%	8%	8.25%	8.5%	8.75%	9%	9.25%
1	1.0725	1.0750	1.0775	1.0800	1.0825	1.0850	1.0875	1.0900	1.0925
2	1.1503	1.1556	1.16101	1.1664	1.1718	1.1772	1.1827	1.1881	1.1936
3	1.2336	1.2423	1.25098	1.2597	1.2685	1.2773	1.2861	1.2950	1.3040
4	1.3231	1.3355	1.34794	1.3605	1.3731	1.3859	1.3987	1.4116	1.4246
5	1.4190	1.4356	1.4524	1.4693	1.4864	1.5037	1.5211	1.5386	1.5563
6	1.5219	1.5433	1.56496	1.5869	1.6090	1.6315	1.6542	1.6771	1.7003
7	1.6322	1.6590	1.68625	1.7138	1.7418	1.7701	1.7989	1.8280	1.8576
8	1.7506	1.7835	1.81693	1.8509	1.8855	1.9206	1.9563	1.9926	2.0294
9	1.8775	1.9172	1.95774	1.9990	2.0410	2.0839	2.1275	2.1719	2.2171
10	2.0136	2.0610	2.10947	2.1589	2.2094	2.2610	2.3136	2.3674	2.4222
11	2.1596	2.2156	2.27295	2.3316	2.3917	2.4532	2.5161	2.5804	2.6463
12	2.3162	2.3818	2.4491	2.5182	2.5890	2.6617	2.7362	2.8127	2.8911
13	2.4841	2.5604	2.63891	2.7196	2.8026	2.8879	2.9756	3.0658	3.1585
14	2.6642	2.7524	2.84343	2.9372	3.0338	3.1334	3.2360	3.3417	3.4506

Tips 1-2-2:

- Use the table (1) if the interest rate or period are in the table.
- If the number of period is greater than 50 years. You should divide the period. **For Example, 5% for 60 years.**

$$(1+5\%)^{60} = (1+5\%)^{50} \times (1+5\%)^{10}$$
$$= 11.4674 \times 1.6289$$
$$= 18.67924$$

- If the interest rate or period are not in the table use the calculator.
- The period in years and months (i.e. n the decimal places)

For Example: 2 years and 3 months = (3/12) + 2 = 2.25

Activity 1-2-1:

1- Find the value of the following (by using the interest table) :

a - $(1.06)^{12}$ b- $(1.0525)^{60}$ c - $(1.005)^{125}$

2- Bader deposited BD4,200 for 14 years at 5.6% annually. Find the future value at the end of the period.

3- A trader wants to borrow BD20,000 and pays it after 3 years, he has two choices:

- Borrowing on a simple interest at 5¾% annually
- Borrowing on a compound interest at 4% annually

Which choice should he choose? Why?

4- Find the future value of BD1,500 at 9.4% annually for 8 years using a calculator.

5- Find the future value and compound interest for BD2,400 at 4.5% annually for 74 years by using interest tables.

6- A person deposited $6000 for 4 years at 5.5% annually. Find the future value and the compound interest at the end of the period.

1.3 Annual and Partial Interest Rate

When you borrow money from a bank, you pay interest, which is a percentage charged on the principle amount for a period of a year or a part of year, like semi-annually, thirdly, quarterly, ...etc. This process is called compounding. In other words, Compounding is the number of adding interest during a year.

For Example: Consider a sum compounded at 12% annually for 5 years for the following Types of compounding:

Important Points 1-3-1:

Annually: (One time a year)

$n = 5$
$i = 12\%$
$(1+ 12\%)^5$

1-Monthly: (Twelfth times a year / Every month)
$n = 5 \times 12 = 60$
$i = 12\% \div 12 = 1\%$
$(1+ 1\%)^{60}$

2-Semi-annually: (Two times a year / Every 6 months)
$n = 5 \times 2 = 10$
$i = 12\% \div 2 = 6\%$
$(1+ 6\%)^{10}$

3-Thirdly: (Three times a year / Every 4 months)
$n = 5 \times 3 = 15$
$i = 12\% \div 3 = 4\%$
$(1+ 4\%)^{15}$

4-Quarterly: (Four times a year / Every 3 months)
$n = 5 \times 4 = 20$
$i = 12\% \div 4 = 3\%$
$(1+ 3\%)^{20}$

Important Notes:

1-Annual interest rate: (12% annually ...etc.)

Find the partial interest rate i = Annual interest rate ÷ No of compounding in a year.

Find the number of periods n = Number of years × No of compounding in a year.

2- Partial interest, also called tenancy in common interest, is **when two or more individuals own a separate portion of an undivided property**. Partial interest in the income approach is a method to determine the value of each owner's share of an income producing property.

A) Annual interest rate

Example 1-3-1:

► Shahd invested BD1000 in a bank at 6% annually for 4 years – find the future value if the interest is compounded annually.

Answer:

The number of times	Once(1) a year n= 1 x 4 years = 4 periods											
months	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
	1	2	3	4	5	6	7	8	9	10	11	12
Rate	i = 6% ÷ 1 = 6%											

$$FV = PV \times (1+i)^n$$

$$= 1000 \times (1+6\%)^4$$

$$= 1000 \times 1.2625 \qquad = BD1262.500$$

Example 1-3-2:

► Shahd invested BD1000 in a bank at 6% annually for 4 years – find the future value if the interest is compounded semi-annually.

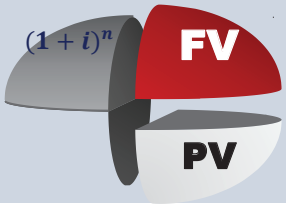
Answer:

The number of times	Twice (2) a year n= 2 x 4 years = 8 periods i = 6% ÷ 2= 3%											
months	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
	1	2	3	4	5	6	7	8	9	10	11	12
Rate	3%						3%					

$$FV = PV \times (1+i)^n$$

$$= 1000 \times (1+3\%)^8$$

$$= 1000 \times 1.2668 \qquad = BD1266.800$$



Example 1-3-3:

► Shahd invested BD1000 in a bank at 6% annually for 4 years – find the future value if the interest is compounded thirdly.

Answer:

The number of times	Three times (3) a year or every four months n= 3 x 4 years = 12 periods i = 6% ÷ 3= 2%											
months	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
	1	2	3	4	5	6	7	8	9	10	11	12
Rate	2%				2%				2%			

$$FV = PV \times (1+i)^n$$
$$= 1000 \times (1+2\%)^{12}$$
$$= 1000 \times 1.2682 = BD1268.200$$

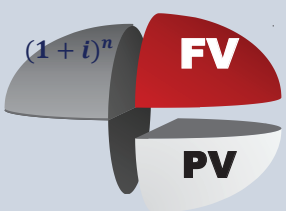
Example 1-3-4:

► Shahd invested BD1000 in a bank at 6% annually for 4 years – find the future value if the interest is compounded quarterly.

Answer:

The number of times	Four times (4) a year or every three months n= 4 x 4 years = 16 periods i = 6% ÷ 4= 1.5%											
months	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
	1	2	3	4	5	6	7	8	9	10	11	12
Rate	1.5%			1.5%			1.5%			1.5%		

$$FV = PV \times (1+i)^n$$
$$= 1000 \times (1+1.5\%)^{16}$$
$$= 1000 \times 1.2690 = BD1269$$



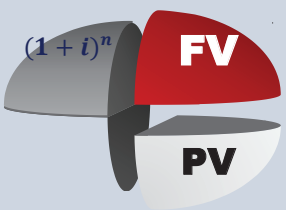
Example 1-3-5:

► Shahd invested BD1000 in a bank at 6% annually for 4 years – find the future value if the interest is compounded monthly.

Answer:

The number of times	Twelve times (12) a year or every month n= 12 x 4 years = 48 periods i = 6% ÷ 12= 0.5%											
months	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
	1	2	3	4	5	6	7	8	9	10	11	12
Rate	½%	½%	½%	½%	½%	½%	½%	½%	½%	½%	½%	½%

$$FV = PV \times (1+i)^n$$
$$= 1000 \times (1+0.5\%)^{48}$$
$$= 1000 \times 1.2705 = \text{BD}1270.500$$



Tips 1-3-1:

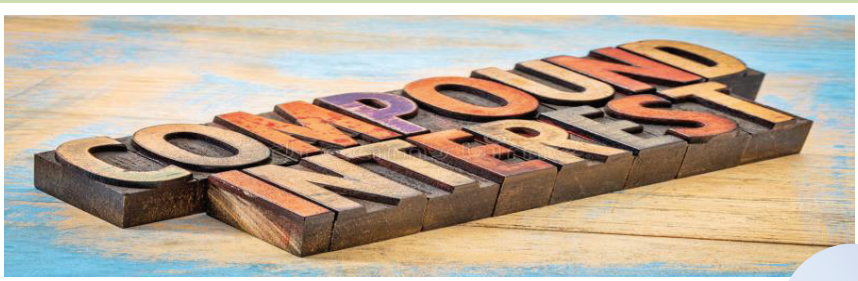
► Annual interest rate: (12% annually ...etc.)

❖ Find the partial interest rate

i = Annual interest rate ÷ No of compounding in a year.

❖ Find the number of periods

n = Number of years ×No of compounding in a year.



B) Partial interest rate

Example 1-3-6:

► A woman deposited BD 5,000 in a bank that pays 3.8% quarterly on saving accounts for 4 years and 6 months. Find the future value.

Answer:

The number of times	Four times (4) a year or every three months n= 4 x 4 years and 6 months = 18 periods											
months	Jan 1	Feb 2	Mar 3	Apr 4	May 5	Jun 6	Jul 7	Aug 8	Sep 9	Oct 10	Nov 11	Dec 12
Rate	3.8%			3.8%			3.8%			3.8%		

$$FV = PV \times (1+i)^n$$
$$= 5000 \times (1+3.8\%)^{18}$$
$$= 5000 \times 1.95682 = BD9784.100$$

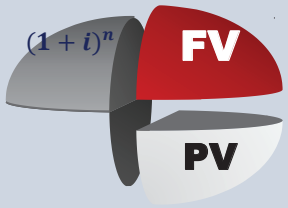
Example 1-3-7:

► Ali deposited BD3000 at 2¾% every 4 months – find the future value at the end of 4 years and 8 months?

Answer:

The number of times	Three times (3) a year or every four months n= 3 x 4 years and 8 months = 14 periods											
months	Jan 1	Feb 2	Mar 3	Apr 4	May 5	Jun 6	Jul 7	Aug 8	Sep 9	Oct 10	Nov 11	Dec 12
Rate	2¾%				2¾%				2¾%			

$$FV = PV \times (1+i)^n$$
$$= 3000 \times (1+2.75\%)^{14}$$
$$= 3000 \times 1.461994 = BD4385.982$$



Tips 1-3-2:

- Partial interest rate: (3% monthly, 3% semiannually ...etc.)
  - Find the number of periods only without changing the rate,

$$(1+3\%)^{60}, (1+3\%)^{10}$$

Activity 1-3-1:

1- Mariam deposited BD2550 in a bank at an effective rate of 6% annually. If the interest is compounded semi-annually. Calculate her fund at the end of 8 years, and then find the compound interest.

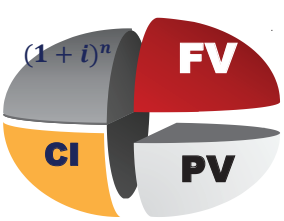
2- Faisal wants to borrow KD7200 from a bank to buy a new car if you know the interest rate is 6% annually compounded monthly. Find how much he will pay to the bank at the end of 10 years.

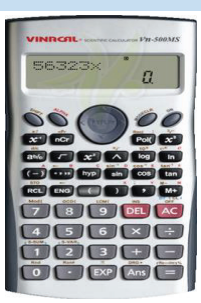
3- Sakeena Ahmed deposited BD2600 in a bank at 12% annually compounded quarterly. Find her fund at the end of 5 years and 9 months and find the compound interest.

4- A person deposited BD8750 at 3% each quarter – find the future value and the interest at the end of 6 years.

5- A person deposited BD3500 at 4% every 6 months. Find the future value at the end of 8 years and 6 months.

6- Laila borrowed $6400 from a bank at 5% every 4 months. Find the amount she will pay at the end of three years and 8 months. In addition, calculate the interest.





1.4 Changeable Interest Rate or Principal

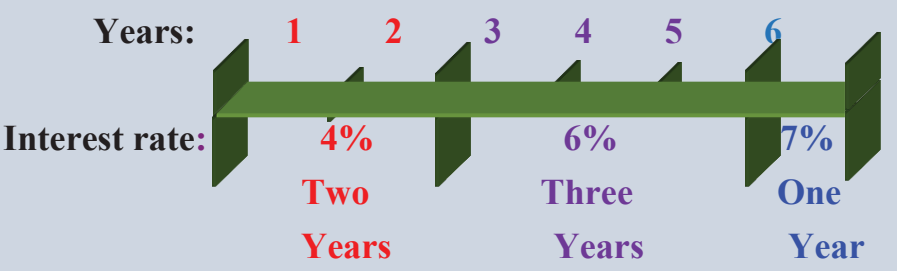
A) Changeable interest rate

The interest rate may change many times throughout the investment life or the borrowing period, in this case raise each interest rate to the power of years that will be used.

Example 1-4-1:

► Hashim invested BD 5,000 for 6 years at 4% annually for the first two years, 6% annually for the following 3 years and 7% annually for the last year; Find the compound interest at the end of the period?

Answer:



$Fv = PV \times (1+i)^n$

$FV = 5000 \times (1+4\%)^2 \times (1+6\%)^3 \times (1+7\%)^1$

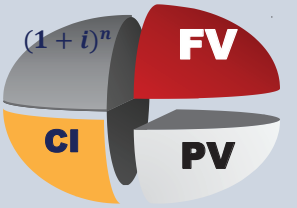
$= 5000 \times 1.0816 \times 1.1910 \times 1.07 = BD\ 6,891.793$

$CI = FV - PV$

$CI = 6,891.793 - 5000 = BD\ 1,891.793$

Or $CI = 5000 \times ([1.0816 \times 1.1910 \times 1.07] - 1)$

$= 5000 \times 0.3783585 = BD\ 1,891.793$



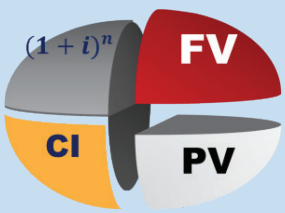
Activity 1-4-1:

1- Find the future value and compound interest for BD10,000 invested for 4 years, if you know that the rate is changing as follows : 3% annually for the first year, 2.5% annually for the second year, 2% annually for the third year and 1% annually for the fourth year.

2- What is the future value for BD2,000 invested at 6% annually for 3 years and 4.8% annually for 5 more years?

3- Find the future value that Saad will get if he saves BD2,000 in a bank for 10 years at changing rate, as following: at 5% annually for the first 5 years, 4.6% annually for the sixth year and 6% annually compounded semi-annually for the rest years.

4- Find the future value for BD2,000 invested for 7 years at 4.5% annually for the first 3 years and 5¾% annually for the rest years.

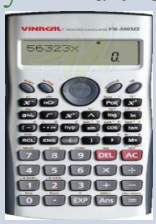


B) Changeable principal

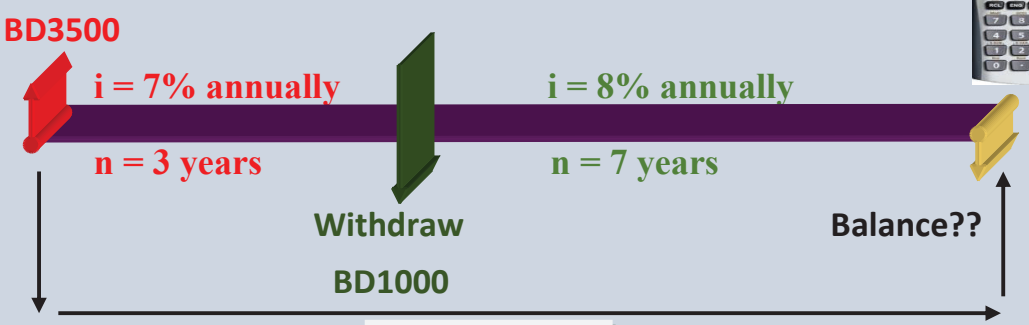
During the investment period, you may add or withdraw an amount of money as well as during the borrowing period. You may pay part of your loan or increase your loan. In these cases, you can calculate your compound amount as illustrated on the following example:

Example 1-4-2:

► Fayez invested BD3500 at 7% annually and after 3 years he withdrew from his account BD1000 and invested the rest at 8% annually - find the future value and the interest at the end of 10 years.



Answer:



CA for the first 3 years

$FV1 = PV1 \times (1+i)^n$

$= 3500 \times (1+7\%)^3$

$= 3500 \times 1.2250$

$= BD\ 4,287.500$

$CI1 = 4287.500 - 3500$

$= BD787.500$

The balance after withdrawal

4,287.500

–

1,000

= 3,287.500

CA for the rest 7 years

$FV2 = PV2 \times (1+i)^n$

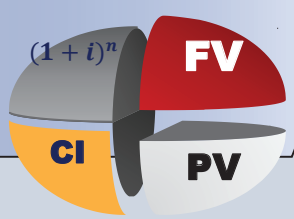
$= 3,287.500 \times (1+8\%)^7$

$= 3,287.500 \times 1.7138$

$= BD\ 5,634.118$

$CI2 = 5634.118 - 3287.500$

$= BD2346.618$



$CI = 787.500 + 2346.618 = BD3134.118$

Or $CI = 5634.118 - (3500 - 1000) = BD3134.118$

Activity 1-4-2:

1- Noora invested BD4000 at Al- Ahli bank at interest rate of 9% annually and after 3 years she deposited BD3000 to her account at 4% every 4 months. Find the future value and the compound interest at the end of 8 years from the first deposit.

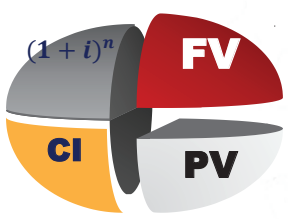
2- A person deposited BD7,000 at 6% annually, after two years he withdrew BD2,618 from his account and invested the rest at 7.5% annually, Find:

a- His fund after withdrawal

b- The future value for the remaining sum at the end of four years.

3- Dawood deposited BD3,000 at a compound interest of 8% annually, after 3 years he added BD1220.864 to his account and the rate of interest increased to 10% annually. Find the future value at the end of 8 years from the first deposit.

4- A trader borrowed BD20,000 on January 1st, 2000 from BBK and BD15,000 on January 1st,2001 and finally BD30,000 on January 2002. He promised to pay the debt on 31/12/ 2008. If the bank gives compound interest rate of 12% annually, find the amount that should be paid on 31/12/2008.



1.5 Finding the Compound Interest Factors

Assume you are planning for a trip after 5 years and it will cost BD2500, how much must you invest to have this amount? To answer this question you need to learn how to find the principal (present value).

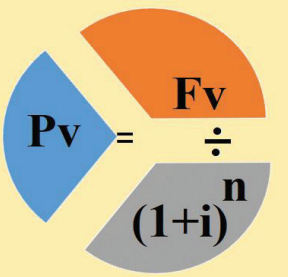
A) Finding the present value (principal) by knowing future value:

To find the present value (principal) by knowing future value:

Important Points 1-5-1:

Present Value = Future Value ÷ (1 + interest rate) $^{\text{Number of periods}}$

$$PV = FV \div (1+i)^n$$



Example 1-5-1:

► Hasan deposited an amount of money in a bank at 5% annually. If the compound amount at the end of 12 years is BD8,081.550. Calculate the present value (principal)?

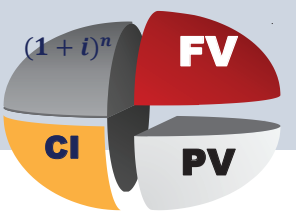
Answer:

$$PV = FV \div (1+i)^n$$

$$PV = 8{,}081.550 \div (1+5\%)^{12}$$

$$= 8{,}081.550 \div 1.7959$$

$$= \text{BD } 4500$$



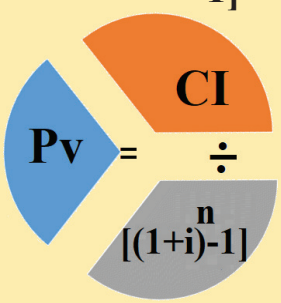
B) Finding the present value (principal) by knowing Compound interest:

To find the unknown present value (principal) by knowing compound interest, we use the following formula:

Important Points 1-5-2:

Present Value = Compound Interest ÷[(1 + interest rate) $^{\text{Number of periods}}$ - 1]

$$PV = CI \div [(1+i)^n - 1]$$



Example 1-5-2:

► A person deposited an amount of money in a bank at 3% annually. If the compound interest at the end of 10 years was BD171.950 – Find:

a) The deposited amount.

b) The compound amount at the end of the period.

Answer:

a) $PV = CI \div [(1+i)^n - 1]$

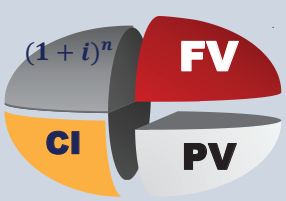
$PV = 171.950 \div [(1+3\%)^{10} - 1]$

$= 171.950 \div [1.3439 - 1]$

$= 171.950 \div 0.3439 = BD\ 500$

b) $FV = PV + CI$

$= 500 + 171.950 = BD\ 671.950$



Activity 1-5-1:

1- Find the present value (principal), if the amount after 5 years is BD1469.330 and the compound interest rate is 8% annually, then find the interest.

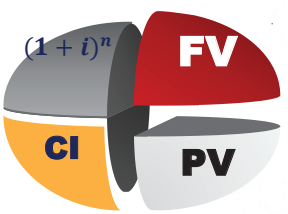
2- Find the present value (principal), if the amount at the end of 7 years is BD6,450 and the compound interest rate is 4.5% annually for the first three years and 5% annually for the rest years.

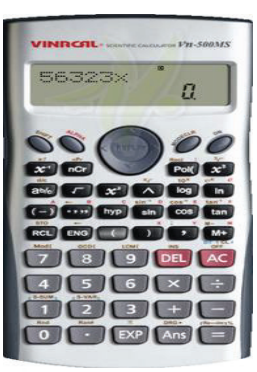
3- How much was deposited for an investment of 8% annually compounded quarterly to have an amount of BD2, 228.850 in 5 years?

4- A man deposited money in NBB at 3%annually, after 5 years he withdrew BD2796.500 from his account and invested the rest for 5 years at 2% every 6 months – if the amount at the end of the period is BD3657. Find the present value.

5- Find the present value (principal) that generates an interest of BD700 at 2% annually for 5 years.

6- Taha calculated the compound interest he will get it if he deposits his money in a bank at 3% annually for 20 years and it was BD 4836.667, find the present value.





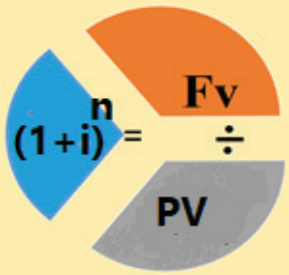
C) Finding the Time/Periods

Investment period is the amount of time an investor is willing to hold an investment. In general, the longer period increases in investment it cause an increase in the interest. To find the unknown time / periods, we use the following formula:

Important Points 1-5-3:

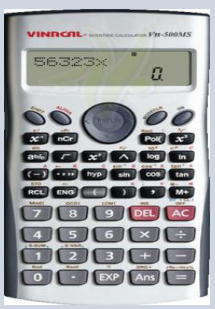
Present Value = Future Value ÷ (1 + interest rate) $^{\text{Number of periods}}$

$$(1+ i)^n = FV \div PV$$



Example 1-5-3:

► A principal of BD3175.309 is invested at 8% annually, how long would it take to make the future value to BD18643.829.



Answer:

$(1+ i)^n = FV \div PV$

$(1+8\%)^n = 18643.829 \div 3175.309$

$= 18643.829 \div 3175.309$

$= 5.8715$ we are choosing to find n = 23 years by two ways.

a)By using calculator

Press log 5.8715 ÷ log 1.08

n = 0.768749065 ÷ 0.033423755

= 23 Years

b) By using interest table No 1:

Searching for 5.8715 under 8%.

N	8%	8.25%	8.5%	8.75%
20	4.6610	4.8816	5.1120	5.3529
21	5.0338	5.2843	5.5466	5.8212
22	5.4365	5.7202	6.0180	6.3306
23	5.8715	6.1922	6.5296	6.8845
24	6.3412	6.7030	7.0846	7.4869

we find n = 23 years.

Example 1-5-4:

How long will it take for BD1600 at 4% every semi-annual to make the amount to BD2561.652?

Answer:

$(1+i)^n = FV \div PV$

$(1+4\%)^n = 2561.652 \div 1600$

$= 2561.652 \div 1600$

$= 1.6010325$ we are choosing to find

n = 12 times ÷ 2 = 6 Years by two ways.

a)By using calculator

Press log 1.60103 ÷ log 1.04

n = 0. 20440147 ÷ 0.017033339

= 12 ÷ 2 = 6 years

b) By using interest table No 1:

Searching for 1.6010 under 4% .

N	3.75%	3.5%	4%	4.25%
9	1.3629	1.3928	1.4233	1.4544
10	1.4106	1.4450	1.4802	1.5162
11	1.4600	1.4992	1.5395	1.5807
12	1.5111	1.5555	1.6010	1.6478
13	1.5640	1.6138	1.6651	1.7179

we find n = 12 ÷ 2 = 6 years.

Activity 1-5-2:

1- A loan of BD2,000 amounted to BD3591.713. Find the borrowing period if the interest rate was 5% annually.

2- Nawal borrowed BD5,000 from a bank at 6% annually. Find the borrowing period if the compound interest was BD 8563.575.

3- How long will it take BD2,000 at 2.5% compounded every semi-annually to give an interest of BD 560.169 ?

4- How long will it take an investment of BD4,000 to amount of BD5,610.400 at 7% annually?

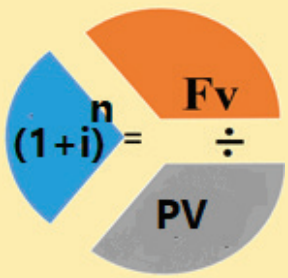
5- How long will it take the money to double itself at 4% annually?

D) Finding the interest rate

Important Points 1-5-3:

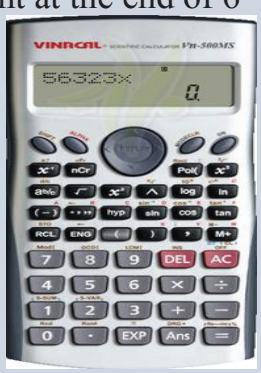
Present Value = Future Value ÷ (1 + interest rate) $^{\text{Number of periods}}$

$$(1+ i)^n = FV \div PV$$



Example 1-5-5:

► Mahmood deposited BD3000 in the bank, if the amount at the end of 6 years was BD5031.300 – Find the interest rate.



Answer:

$(1+ i)^n = FV \div PV$

$(1+i\%)^6 = 5031.300 \div 3000$

$= 1.6771$ we finding the time by two ways.

I = 9% Annually.

a) By using calculator

Press $\boxed{6}\ \boxed{x}\ \boxed{\sqrt{}}$ 1.6771 = 1.08999

i =1.80999 –1= 0.8999 x 100 = 9%

b) By using interest table No 1

Searching for 1.6771 opposite to n = 6.

N	8.75%	8.5%	9%	9.25%
3	1.2773	1.2861	1.2950	1.3040
4	1.3859	1.3987	1.4116	1.4246
5	1.5037	1.5211	1.5386	1.5563
6	1.6315	1.6542	1.6771	1.7003
7	1.7701	1.7989	1.8280	1.8576

We Find the i = 9%

Activity 1-5-3:

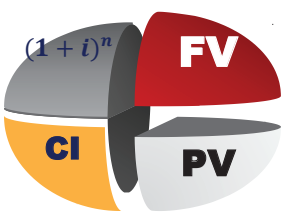
1- Saad deposited BD1,400 in a bank that gives compound interest of BD 481.460 at the end of 10 years – Find the interest rate.

2- Tahera borrowed €4,500 from a bank that gives compound interest of €1,916 at the end of 3 years. Find the quarterly interest rate and the annual rate of interest.

3- If BD 6,600 amounts to BD10,750.740 in 10 years. Find the interest rate.

4- Jawad borrowed BD8,000 for 4 years. If the compound interest was BD 2,948.800 find the semi-annually interest rate, and the annually interest rate.

5- Find the interest rate for ¥7,730.325 amounts to ¥10,000 after 13 years.



Exercises

1. Find the future value and interest of BD2500 invested in a bank, at the end of 20 years at 5.25% annually.
2. Find the future value and compound interest for BD1500 invested at 6.25% annually for 25 years.
3. Find the future value for BD8,750 invested for 10 years, if you know that the rate is changing as follows: 5%annually for the first 5 years, 5.25% annually for the sixth year, 5.5% annually for the remaining years.
4. Ahmed invested BD7,400 at 6.5% annually, find the future value after 8.5 years.
5. Moayed deposited BD1400 at 6% annually, find the future value at the end of 10 years and 8 months if you know that interest is compounded quarterly.
6. Shahd invested BD6000 at 10% annually compounded semi- annually . Find the future value at the end of 4 years and 5 months.
7. Malak invested BD3150 at 2¼% every semi – annual, find the future value at the end of 4 years and 7 months.
8. Omar borrowed BD6,000 from BBK for 5 years at 4%annually compounded semi-annually. Two years including the borrowing date, he borrowed BD2,980.600 at 2% annually compounded quarterly. How much would he will pay after 5 years from first borrowed date?
9. How long will it take the money to triple itself at 6.75% annually?
10. Jassim deposited money in BBK for investment at 5% annually, after 5 years he added BD 723.718 to his account at a new interest rate of 5.5% annually. After 3 years, his account was BD2153.781. Find the first sum deposited.

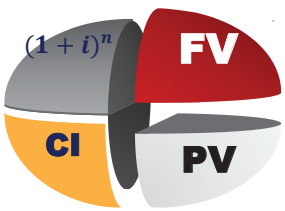


TABLE No.1	Equation : $FV = PV \times (1 + i)^n$						FV for one monetary unit		
n i	0.5%	0.75%	1%	1.25%	1.5%	1.75%	2%	2.25%	2.5%
1	1.0050	1.0075	1.0100	1.0125	1.0150	1.0175	1.0200	1.0225	1.0250
2	1.0100	1.0151	1.0201	1.0252	1.0302	1.0353	1.0404	1.0455	1.0506
3	1.0151	1.0227	1.0303	1.0380	1.0457	1.0534	1.0612	1.0690	1.0769
4	1.0202	1.0303	1.0406	1.0509	1.0614	1.0719	1.0824	1.0931	1.1038
5	1.0253	1.0381	1.0510	1.0641	1.0773	1.0906	1.1041	1.1177	1.1314
6	1.0304	1.0459	1.0615	1.0774	1.0934	1.1097	1.1262	1.1428	1.1597
7	1.0355	1.0537	1.0721	1.0909	1.1098	1.1291	1.1487	1.1685	1.1887
8	1.0407	1.0616	1.0829	1.1045	1.1265	1.1489	1.1717	1.1948	1.2184
9	1.0459	1.0696	1.0937	1.1183	1.1434	1.1690	1.1951	1.2217	1.2489
10	1.0511	1.0776	1.1046	1.1323	1.1605	1.1894	1.2190	1.2492	1.2801
11	1.0564	1.0857	1.1157	1.1464	1.1779	1.2103	1.2434	1.2773	1.3121
12	1.0617	1.0938	1.1268	1.1608	1.1956	1.2314	1.2682	1.3060	1.3449
13	1.0670	1.1020	1.1381	1.1753	1.2136	1.2530	1.2936	1.3354	1.3785
14	1.0723	1.1103	1.1495	1.1900	1.2318	1.2749	1.3195	1.3655	1.4130
15	1.0777	1.1186	1.1610	1.2048	1.2502	1.2972	1.3459	1.3962	1.4483
16	1.0831	1.1270	1.1726	1.2199	1.2690	1.3199	1.3728	1.4276	1.4845
17	1.0885	1.1354	1.1843	1.2351	1.2880	1.3430	1.4002	1.4597	1.5216
18	1.0939	1.1440	1.1961	1.2506	1.3073	1.3665	1.4282	1.4926	1.5597
19	1.0994	1.1525	1.2081	1.2662	1.3270	1.3904	1.4568	1.5262	1.5987
20	1.1049	1.1612	1.2202	1.2820	1.3469	1.4148	1.4859	1.5605	1.6386
21	1.1104	1.1699	1.2324	1.2981	1.3671	1.4395	1.5157	1.5956	1.6796
22	1.1160	1.1787	1.2447	1.3143	1.3876	1.4647	1.5460	1.6315	1.7216
23	1.1216	1.1875	1.2572	1.3307	1.4084	1.4904	1.5769	1.6682	1.7646
24	1.1272	1.1964	1.2697	1.3474	1.4295	1.5164	1.6084	1.7058	1.8087
25	1.1328	1.2054	1.2824	1.3642	1.4509	1.5430	1.6406	1.7441	1.8539
26	1.1385	1.2144	1.2953	1.3812	1.4727	1.5700	1.6734	1.7834	1.9003
27	1.1442	1.2235	1.3082	1.3985	1.4948	1.5975	1.7069	1.8235	1.9478
28	1.1499	1.2327	1.3213	1.4160	1.5172	1.6254	1.7410	1.8645	1.9965
29	1.1556	1.2420	1.3345	1.4337	1.5400	1.6539	1.7758	1.9065	2.0464
30	1.1614	1.2513	1.3478	1.4516	1.5631	1.6828	1.8114	1.9494	2.0976
31	1.1672	1.2607	1.3613	1.4698	1.5865	1.7122	1.8476	1.9933	2.1500
32	1.1730	1.2701	1.3749	1.4881	1.6103	1.7422	1.8845	2.0381	2.2038
33	1.1789	1.2796	1.3887	1.5067	1.6345	1.7727	1.9222	2.0840	2.2589
34	1.1848	1.2892	1.4026	1.5256	1.6590	1.8037	1.9607	2.1308	2.3153
35	1.1907	1.2989	1.4166	1.5446	1.6839	1.8353	1.9999	2.1788	2.3732
36	1.1967	1.3086	1.4308	1.5639	1.7091	1.8674	2.0399	2.2278	2.4325
37	1.2027	1.3185	1.4451	1.5835	1.7348	1.9001	2.0807	2.2779	2.4933
38	1.2087	1.3283	1.4595	1.6033	1.7608	1.9333	2.1223	2.3292	2.5557
39	1.2147	1.3383	1.4741	1.6233	1.7872	1.9672	2.1647	2.3816	2.6196
40	1.2208	1.3483	1.4889	1.6436	1.8140	2.0016	2.2080	2.4352	2.6851
41	1.2269	1.3585	1.5038	1.6642	1.8412	2.0366	2.2522	2.4900	2.7522
42	1.2330	1.3686	1.5188	1.6850	1.8688	2.0723	2.2972	2.5460	2.8210
43	1.2392	1.3789	1.5340	1.7060	1.8969	2.1085	2.3432	2.6033	2.8915
44	1.2454	1.3893	1.5493	1.7274	1.9253	2.1454	2.3901	2.6619	2.9638
45	1.2516	1.3997	1.5648	1.7489	1.9542	2.1830	2.4379	2.7218	3.0379
46	1.2579	1.4102	1.5805	1.7708	1.9835	2.2212	2.4866	2.7830	3.1139
47	1.2642	1.4207	1.5963	1.7929	2.0133	2.2600	2.5363	2.8456	3.1917
48	1.2705	1.4314	1.6122	1.8154	2.0435	2.2996	2.5871	2.9096	3.2715
49	1.2768	1.4421	1.6283	1.8380	2.0741	2.3398	2.6388	2.9751	3.3533
50	1.2832	1.4530	1.6446	1.8610	2.1052	2.3808	2.6916	3.0420	3.4371

TABLE No.1	Equation : $FV = PV \times (1 + i)^n$					FV for one monetary unit			
n i	2.75%	3%	3.25%	3.5%	3.75%	4%	4.25%	4.5%	4.75%
1	1.0275	1.0300	1.0325	1.0350	1.0375	1.0400	1.0425	1.0450	1.0475
2	1.0558	1.0609	1.0661	1.0712	1.0764	1.0816	1.0868	1.0920	1.0973
3	1.0848	1.0927	1.1007	1.1087	1.1168	1.1249	1.1330	1.1412	1.1494
4	1.1146	1.1255	1.1365	1.1475	1.1587	1.1699	1.1811	1.1925	1.2040
5	1.1453	1.1593	1.1734	1.1877	1.2021	1.2167	1.2313	1.2462	1.2612
6	1.1768	1.1941	1.2115	1.2293	1.2472	1.2653	1.2837	1.3023	1.3211
7	1.2091	1.2299	1.2509	1.2723	1.2939	1.3159	1.3382	1.3609	1.3838
8	1.2424	1.2668	1.2916	1.3168	1.3425	1.3686	1.3951	1.4221	1.4495
9	1.2765	1.3048	1.3336	1.3629	1.3928	1.4233	1.4544	1.4861	1.5184
10	1.3117	1.3439	1.3769	1.4106	1.4450	1.4802	1.5162	1.5530	1.5905
11	1.3477	1.3842	1.4216	1.4600	1.4992	1.5395	1.5807	1.6229	1.6661
12	1.3848	1.4258	1.4678	1.5111	1.5555	1.6010	1.6478	1.6959	1.7452
13	1.4229	1.4685	1.5156	1.5640	1.6138	1.6651	1.7179	1.7722	1.8281
14	1.4620	1.5126	1.5648	1.6187	1.6743	1.7317	1.7909	1.8519	1.9149
15	1.5022	1.5580	1.6157	1.6753	1.7371	1.8009	1.8670	1.9353	2.0059
16	1.5435	1.6047	1.6682	1.7340	1.8022	1.8730	1.9463	2.0224	2.1012
17	1.5860	1.6528	1.7224	1.7947	1.8698	1.9479	2.0291	2.1134	2.2010
18	1.6296	1.7024	1.7784	1.8575	1.9399	2.0258	2.1153	2.2085	2.3055
19	1.6744	1.7535	1.8362	1.9225	2.0127	2.1068	2.2052	2.3079	2.4151
20	1.7204	1.8061	1.8958	1.9898	2.0882	2.1911	2.2989	2.4117	2.5298
21	1.7677	1.8603	1.9575	2.0594	2.1665	2.2788	2.3966	2.5202	2.6499
22	1.8164	1.9161	2.0211	2.1315	2.2477	2.3699	2.4985	2.6337	2.7758
23	1.8663	1.9736	2.0868	2.2061	2.3320	2.4647	2.6047	2.7522	2.9077
24	1.9176	2.0328	2.1546	2.2833	2.4194	2.5633	2.7153	2.8760	3.0458
25	1.9704	2.0938	2.2246	2.3632	2.5102	2.6658	2.8308	3.0054	3.1904
26	2.0245	2.1566	2.2969	2.4460	2.6043	2.7725	2.9511	3.1407	3.3420
27	2.0802	2.2213	2.3715	2.5316	2.7020	2.8834	3.0765	3.2820	3.5007
28	2.1374	2.2879	2.4486	2.6202	2.8033	2.9987	3.2072	3.4297	3.6670
29	2.1962	2.3566	2.5282	2.7119	2.9084	3.1187	3.3435	3.5840	3.8412
30	2.2566	2.4273	2.6104	2.8068	3.0175	3.2434	3.4856	3.7453	4.0237
31	2.3187	2.5001	2.6952	2.9050	3.1306	3.3731	3.6338	3.9139	4.2148
32	2.3824	2.5751	2.7828	3.0067	3.2480	3.5081	3.7882	4.0900	4.4150
33	2.4479	2.6523	2.8732	3.1119	3.3698	3.6484	3.9492	4.2740	4.6247
34	2.5153	2.7319	2.9666	3.2209	3.4962	3.7943	4.1171	4.4664	4.8444
35	2.5844	2.8139	3.0630	3.3336	3.6273	3.9461	4.2920	4.6673	5.0745
36	2.6555	2.8983	3.1626	3.4503	3.7633	4.1039	4.4744	4.8774	5.3155
37	2.7285	2.9852	3.2654	3.5710	3.9045	4.2681	4.6646	5.0969	5.5680
38	2.8036	3.0748	3.3715	3.6960	4.0509	4.4388	4.8628	5.3262	5.8325
39	2.8807	3.1670	3.4811	3.8254	4.2028	4.6164	5.0695	5.5659	6.1095
40	2.9599	3.2620	3.5942	3.9593	4.3604	4.8010	5.2850	5.8164	6.3997
41	3.0413	3.3599	3.7110	4.0978	4.5239	4.9931	5.5096	6.0781	6.7037
42	3.1249	3.4607	3.8316	4.2413	4.6935	5.1928	5.7437	6.3516	7.0221
43	3.2108	3.5645	3.9561	4.3897	4.8695	5.4005	5.9878	6.6374	7.3557
44	3.2991	3.6715	4.0847	4.5433	5.0522	5.6165	6.2423	6.9361	7.7051
45	3.3899	3.7816	4.2175	4.7024	5.2416	5.8412	6.5076	7.2482	8.0711
46	3.4831	3.8950	4.3545	4.8669	5.4382	6.0748	6.7842	7.5744	8.4545
47	3.5789	4.0119	4.4961	5.0373	5.6421	6.3178	7.0725	7.9153	8.8560
48	3.6773	4.1323	4.6422	5.2136	5.8537	6.5705	7.3731	8.2715	9.2767
49	3.7784	4.2562	4.7931	5.3961	6.0732	6.8333	7.6865	8.6437	9.7173
50	3.8823	4.3839	4.9488	5.5849	6.3009	7.1067	8.0131	9.0326	10.1789

TABLE No.1	Equation : $FV = PV \times (1 + i)^n$					FV for one monetary unit			
n i	5%	5.25%	5.5%	5.75%	6%	6.25%	6.5%	6.75%	7%
1	1.0500	1.0525	1.0550	1.0575	1.0600	1.0625	1.0650	1.0675	1.0700
2	1.1025	1.1078	1.1130	1.1183	1.1236	1.1289	1.1342	1.1396	1.1449
3	1.1576	1.1659	1.1742	1.1826	1.1910	1.1995	1.2079	1.2165	1.2250
4	1.2155	1.2271	1.2388	1.2506	1.2625	1.2744	1.2865	1.2986	1.3108
5	1.2763	1.2915	1.3070	1.3225	1.3382	1.3541	1.3701	1.3862	1.4026
6	1.3401	1.3594	1.3788	1.3986	1.4185	1.4387	1.4591	1.4798	1.5007
7	1.4071	1.4307	1.4547	1.4790	1.5036	1.5286	1.5540	1.5797	1.6058
8	1.4775	1.5058	1.5347	1.5640	1.5938	1.6242	1.6550	1.6863	1.7182
9	1.5513	1.5849	1.6191	1.6540	1.6895	1.7257	1.7626	1.8002	1.8385
10	1.6289	1.6681	1.7081	1.7491	1.7908	1.8335	1.8771	1.9217	1.9672
11	1.7103	1.7557	1.8021	1.8496	1.8983	1.9481	1.9992	2.0514	2.1049
12	1.7959	1.8478	1.9012	1.9560	2.0122	2.0699	2.1291	2.1899	2.2522
13	1.8856	1.9449	2.0058	2.0684	2.1329	2.1993	2.2675	2.3377	2.4098
14	1.9799	2.0470	2.1161	2.1874	2.2609	2.3367	2.4149	2.4955	2.5785
15	2.0789	2.1544	2.2325	2.3132	2.3966	2.4828	2.5718	2.6639	2.7590
16	2.1829	2.2675	2.3553	2.4462	2.5404	2.6379	2.7390	2.8437	2.9522
17	2.2920	2.3866	2.4848	2.5868	2.6928	2.8028	2.9170	3.0357	3.1588
18	2.4066	2.5119	2.6215	2.7356	2.8543	2.9780	3.1067	3.2406	3.3799
19	2.5270	2.6437	2.7656	2.8929	3.0256	3.1641	3.3086	3.4593	3.6165
20	2.6533	2.7825	2.9178	3.0592	3.2071	3.3619	3.5236	3.6928	3.8697
21	2.7860	2.9286	3.0782	3.2351	3.3996	3.5720	3.7527	3.9421	4.1406
22	2.9253	3.0824	3.2475	3.4211	3.6035	3.7952	3.9966	4.2082	4.4304
23	3.0715	3.2442	3.4262	3.6178	3.8197	4.0324	4.2564	4.4922	4.7405
24	3.2251	3.4145	3.6146	3.8259	4.0489	4.2844	4.5331	4.7954	5.0724
25	3.3864	3.5938	3.8134	4.0458	4.2919	4.5522	4.8277	5.1191	5.4274
26	3.5557	3.7825	4.0231	4.2785	4.5494	4.8367	5.1415	5.4647	5.8074
27	3.7335	3.9810	4.2444	4.5245	4.8223	5.1390	5.4757	5.8335	6.2139
28	3.9201	4.1900	4.4778	4.7847	5.1117	5.4602	5.8316	6.2273	6.6488
29	4.1161	4.4100	4.7241	5.0598	5.4184	5.8015	6.2107	6.6477	7.1143
30	4.3219	4.6416	4.9840	5.3507	5.7435	6.1641	6.6144	7.0964	7.6123
31	4.5380	4.8852	5.2581	5.6584	6.0881	6.5493	7.0443	7.5754	8.1451
32	4.7649	5.1417	5.5473	5.9837	6.4534	6.9587	7.5022	8.0867	8.7153
33	5.0032	5.4116	5.8524	6.3278	6.8406	7.3936	7.9898	8.6326	9.3253
34	5.2533	5.6958	6.1742	6.6916	7.2510	7.8557	8.5092	9.2153	9.9781
35	5.5160	5.9948	6.5138	7.0764	7.6861	8.3467	9.0623	9.8373	10.6766
36	5.7918	6.3095	6.8721	7.4833	8.1473	8.8683	9.6513	10.5013	11.4239
37	6.0814	6.6408	7.2501	7.9136	8.6361	9.4226	10.2786	11.2102	12.2236
38	6.3855	6.9894	7.6488	8.3686	9.1543	10.0115	10.9467	11.9668	13.0793
39	6.7048	7.3563	8.0695	8.8498	9.7035	10.6372	11.6583	12.7746	13.9948
40	7.0400	7.7426	8.5133	9.3587	10.2857	11.3021	12.4161	13.6369	14.9745
41	7.3920	8.1490	8.9815	9.8968	10.9029	12.0084	13.2231	14.5574	16.0227
42	7.7616	8.5769	9.4755	10.4659	11.5570	12.7590	14.0826	15.5400	17.1443
43	8.1497	9.0271	9.9967	11.0677	12.2505	13.5564	14.9980	16.5890	18.3444
44	8.5572	9.5011	10.5465	11.7041	12.9855	14.4037	15.9729	17.7087	19.6285
45	8.9850	9.9999	11.1266	12.3770	13.7646	15.3039	17.0111	18.9040	21.0025
46	9.4343	10.5249	11.7385	13.0887	14.5905	16.2604	18.1168	20.1801	22.4726
47	9.9060	11.0774	12.3841	13.8413	15.4659	17.2767	19.2944	21.5422	24.0457
48	10.4013	11.6590	13.0653	14.6372	16.3939	18.3565	20.5485	22.9963	25.7289
49	10.9213	12.2711	13.7838	15.4788	17.3775	19.5037	21.8842	24.5486	27.5299
50	11.4674	12.9153	14.5420	16.3689	18.4202	20.7227	23.3067	26.2056	29.4570

TABLE No.1	Equation : $FV = PV \times (1 + i)^n$						FV for one monetary unit		
n i	7.25%	7.5%	7.75%	8%	8.25%	8.5%	8.75%	9%	9.25%
1	1.0725	1.0750	1.0775	1.0800	1.0825	1.0850	1.0875	1.0900	1.0925
2	1.1503	1.1556	1.16101	1.1664	1.1718	1.1772	1.1827	1.1881	1.1936
3	1.2336	1.2423	1.25098	1.2597	1.2685	1.2773	1.2861	1.2950	1.3040
4	1.3231	1.3355	1.34794	1.3605	1.3731	1.3859	1.3987	1.4116	1.4246
5	1.4190	1.4356	1.4524	1.4693	1.4864	1.5037	1.5211	1.5386	1.5563
6	1.5219	1.5433	1.56496	1.5869	1.6090	1.6315	1.6542	1.6771	1.7003
7	1.6322	1.6590	1.68625	1.7138	1.7418	1.7701	1.7989	1.8280	1.8576
8	1.7506	1.7835	1.81693	1.8509	1.8855	1.9206	1.9563	1.9926	2.0294
9	1.8775	1.9172	1.95774	1.9990	2.0410	2.0839	2.1275	2.1719	2.2171
10	2.0136	2.0610	2.10947	2.1589	2.2094	2.2610	2.3136	2.3674	2.4222
11	2.1596	2.2156	2.27295	2.3316	2.3917	2.4532	2.5161	2.5804	2.6463
12	2.3162	2.3818	2.4491	2.5182	2.5890	2.6617	2.7362	2.8127	2.8911
13	2.4841	2.5604	2.63891	2.7196	2.8026	2.8879	2.9756	3.0658	3.1585
14	2.6642	2.7524	2.84343	2.9372	3.0338	3.1334	3.2360	3.3417	3.4506
15	2.8573	2.9589	3.06379	3.1722	3.2841	3.3997	3.5192	3.6425	3.7698
16	3.0645	3.1808	3.30124	3.4259	3.5551	3.6887	3.8271	3.9703	4.1185
17	3.2867	3.4194	3.55708	3.7000	3.8483	4.0023	4.1620	4.3276	4.4995
18	3.5249	3.6758	3.83275	3.9960	4.1658	4.3425	4.5261	4.7171	4.9157
19	3.7805	3.9515	4.12979	4.3157	4.5095	4.7116	4.9222	5.1417	5.3704
20	4.0546	4.2479	4.44985	4.6610	4.8816	5.1120	5.3529	5.6044	5.8672
21	4.3485	4.5664	4.79472	5.0338	5.2843	5.5466	5.8212	6.1088	6.4099
22	4.6638	4.9089	5.16631	5.4365	5.7202	6.0180	6.3306	6.6586	7.0028
23	5.0019	5.2771	5.56669	5.8715	6.1922	6.5296	6.8845	7.2579	7.6506
24	5.3646	5.6729	5.99811	6.3412	6.7030	7.0846	7.4869	7.9111	8.3582
25	5.7535	6.0983	6.46297	6.8485	7.2560	7.6868	8.1420	8.6231	9.1314
26	6.1706	6.5557	6.96385	7.3964	7.8546	8.3401	8.8544	9.3992	9.9760
27	6.6180	7.0474	7.50355	7.9881	8.5026	9.0490	9.6292	10.2451	10.8988
28	7.0978	7.5759	8.08507	8.6271	9.2041	9.8182	10.4718	11.1671	11.9069
29	7.6124	8.1441	8.71166	9.3173	9.9634	10.6528	11.3880	12.1722	13.0083
30	8.1643	8.7550	9.38682	10.0627	10.7854	11.5583	12.3845	13.2677	14.2116
31	8.7562	9.4116	10.1143	10.8677	11.6752	12.5407	13.4681	14.4618	15.5262
32	9.3910	10.1174	10.8982	11.7371	12.6384	13.6067	14.6466	15.7633	16.9624
33	10.0719	10.8763	11.7428	12.6760	13.6811	14.7632	15.9282	17.1820	18.5314
34	10.8021	11.6920	12.6528	13.6901	14.8098	16.0181	17.3219	18.7284	20.2455
35	11.5853	12.5689	13.6334	14.7853	16.0316	17.3796	18.8375	20.4140	22.1182
36	12.4252	13.5115	14.69	15.9682	17.3542	18.8569	20.4858	22.2512	24.1642
37	13.3260	14.5249	15.8285	17.2456	18.7859	20.4597	22.2783	24.2538	26.3994
38	14.2921	15.6143	17.0552	18.6253	20.3358	22.1988	24.2277	26.4367	28.8413
39	15.3283	16.7853	18.377	20.1153	22.0135	24.0857	26.3476	28.8160	31.5091
40	16.4396	18.0442	19.8012	21.7245	23.8296	26.1330	28.6530	31.4094	34.4237
41	17.6315	19.3976	21.3358	23.4625	25.7955	28.3543	31.1602	34.2363	37.6079
42	18.9098	20.8524	22.9893	25.3395	27.9236	30.7644	33.8867	37.3175	41.0866
43	20.2807	22.4163	24.771	27.3666	30.2273	33.3794	36.8518	40.6761	44.8872
44	21.7511	24.0975	26.6907	29.5560	32.7211	36.2167	40.0763	44.3370	49.0392
45	23.3281	25.9048	28.7592	31.9204	35.4206	39.2951	43.5830	48.3273	53.5754
46	25.0193	27.8477	30.9881	34.4741	38.3428	42.6352	47.3965	52.6767	58.5311
47	26.8332	29.9363	33.3897	37.2320	41.5061	46.2592	51.5437	57.4176	63.9452
48	28.7787	32.1815	35.9774	40.2106	44.9303	50.1912	56.0538	62.5852	69.8601
49	30.8651	34.5951	38.7656	43.4274	48.6371	54.4574	60.9585	68.2179	76.3222
50	33.1028	37.1897	41.7699	46.9016	52.6496	59.0863	66.2923	74.3575	83.3820

TABLE No.1	Equation : $FV = PV \times (1 + i)^n$						FV for one monetary unit		
n i	9.5%	9.75%	10%	10.25%	10.5%	10.75%	11%	11.25%	11.5%
1	1.0950	1.0975	1.1000	1.1025	1.1050	1.1075	1.1100	1.1125	1.1150
2	1.1990	1.2045	1.2100	1.2155	1.2210	1.2266	1.2321	1.2377	1.2432
3	1.3129	1.3219	1.3310	1.3401	1.3492	1.3584	1.3676	1.3769	1.3862
4	1.4377	1.4508	1.4641	1.4775	1.4909	1.5044	1.5181	1.5318	1.5456
5	1.5742	1.5923	1.6105	1.6289	1.6474	1.6662	1.6851	1.7041	1.7234
6	1.7238	1.7475	1.7716	1.7959	1.8204	1.8453	1.8704	1.8958	1.9215
7	1.8876	1.9179	1.9487	1.9799	2.0116	2.0436	2.0762	2.1091	2.1425
8	2.0669	2.1049	2.1436	2.1829	2.2228	2.2633	2.3045	2.3464	2.3889
9	2.2632	2.3102	2.3579	2.4066	2.4562	2.5066	2.5580	2.6104	2.6636
10	2.4782	2.5354	2.5937	2.6533	2.7141	2.7761	2.8394	2.9040	2.9699
11	2.7137	2.7826	2.8531	2.9253	2.9991	3.0745	3.1518	3.2307	3.3115
12	2.9715	3.0539	3.1384	3.2251	3.3140	3.4051	3.4985	3.5942	3.6923
13	3.2537	3.3517	3.4523	3.5557	3.6619	3.7711	3.8833	3.9985	4.1169
14	3.5629	3.6784	3.7975	3.9201	4.0464	4.1765	4.3104	4.4484	4.5904
15	3.9013	4.0371	4.1772	4.3219	4.4713	4.6255	4.7846	4.9488	5.1183
16	4.2719	4.4307	4.5950	4.7649	4.9408	5.1227	5.3109	5.5055	5.7069
17	4.6778	4.8627	5.0545	5.2533	5.4596	5.6734	5.8951	6.1249	6.3632
18	5.1222	5.3368	5.5599	5.7918	6.0328	6.2833	6.5436	6.8140	7.0949
19	5.6088	5.8571	6.1159	6.3855	6.6663	6.9587	7.2633	7.5805	7.9108
20	6.1416	6.4282	6.7275	7.0400	7.3662	7.7068	8.0623	8.4334	8.8206
21	6.7251	7.0550	7.4002	7.7616	8.1397	8.5353	8.9492	9.3821	9.8350
22	7.3639	7.7428	8.1403	8.5572	8.9944	9.4528	9.9336	10.4376	10.9660
23	8.0635	8.4978	8.9543	9.4343	9.9388	10.4690	11.0263	11.6118	12.2271
24	8.8296	9.3263	9.8497	10.4013	10.9823	11.5944	12.2392	12.9182	13.6332
25	9.6684	10.2356	10.8347	11.4674	12.1355	12.8408	13.5855	14.3714	15.2010
26	10.5869	11.2336	11.9182	12.6428	13.4097	14.2212	15.0799	15.9882	16.9491
27	11.5926	12.3288	13.1100	13.9387	14.8177	15.7500	16.7386	17.7869	18.8982
28	12.6939	13.5309	14.4210	15.3674	16.3736	17.4431	18.5799	19.7879	21.0715
29	13.8998	14.8502	15.8631	16.9426	18.0928	19.3183	20.6237	22.0141	23.4948
30	15.2203	16.2981	17.4494	18.6792	19.9926	21.3950	22.8923	24.4907	26.1967
31	16.6662	17.8871	19.1943	20.5938	22.0918	23.6949	25.4104	27.2459	29.2093
32	18.2495	19.6311	21.1138	22.7047	24.4114	26.2422	28.2056	30.3110	32.5683
33	19.9832	21.5451	23.2252	25.0319	26.9746	29.0632	31.3082	33.7210	36.3137
34	21.8816	23.6458	25.5477	27.5977	29.8069	32.1875	34.7521	37.5146	40.4898
35	23.9604	25.9513	28.1024	30.4264	32.9367	35.6476	38.5749	41.7350	45.1461
36	26.2366	28.4815	30.9127	33.5451	36.3950	39.4798	42.8181	46.4302	50.3379
37	28.7291	31.2585	34.0039	36.9835	40.2165	43.7238	47.5281	51.6536	56.1268
38	31.4584	34.3062	37.4043	40.7743	44.4392	48.4241	52.7562	57.4646	62.5814
39	34.4469	37.6510	41.1448	44.9537	49.1054	53.6297	58.5593	63.9294	69.7782
40	37.7194	41.3220	45.2593	49.5614	54.2614	59.3949	65.0009	71.1215	77.8027
41	41.3027	45.3509	49.7852	54.6415	59.9589	65.7799	72.1510	79.1226	86.7500
42	45.2265	49.7726	54.7637	60.2422	66.2545	72.8512	80.0876	88.0239	96.7263
43	49.5230	54.6254	60.2401	66.4171	73.2113	80.6827	88.8972	97.9266	107.8498
44	54.2277	59.9514	66.2641	73.2248	80.8985	89.3561	98.6759	108.9434	120.2525
45	59.3793	65.7967	72.8905	80.7304	89.3928	98.9619	109.5302	121.1995	134.0816
46	65.0204	72.2118	80.1795	89.0052	98.7790	109.6003	121.5786	134.8345	149.5009
47	71.1973	79.2525	88.1975	98.1283	109.1508	121.3823	134.9522	150.0033	166.6935
48	77.9611	86.9796	97.0172	108.1860	120.6117	134.4310	149.7970	166.8787	185.8633
49	85.3674	95.4601	106.7190	119.2760	133.2759	148.8823	166.2746	185.6526	207.2376
50	93.4773	104.7670	117.3910	131.5010	147.2699	164.8871	184.5648	206.5385	231.0699

Unit 2

Annuities & Amortization Loan

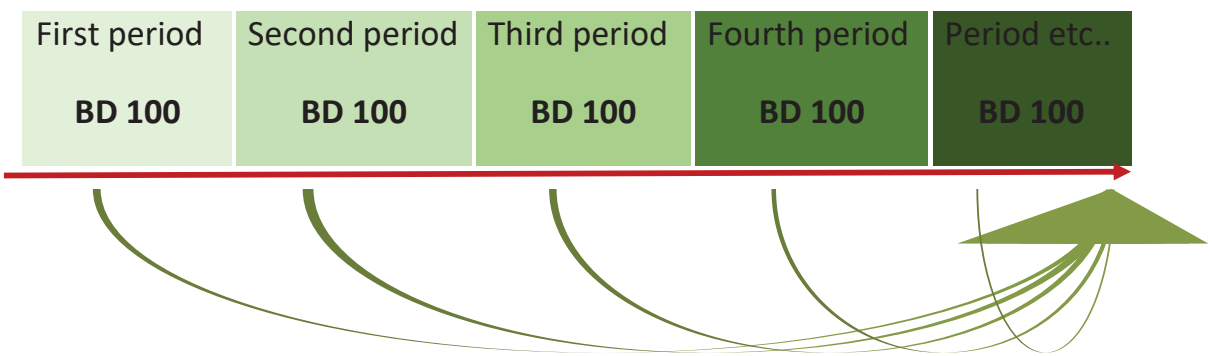
Learning Objective

At this unit, our students should will learn:

- The concept of annuity/ payments
- The different types of annuities
- The definition of future and present value of annuities.
- The calculation of the future and present value of annuities.
- The calculation of the value of annuity.
- The preparation of amortized loan schedule

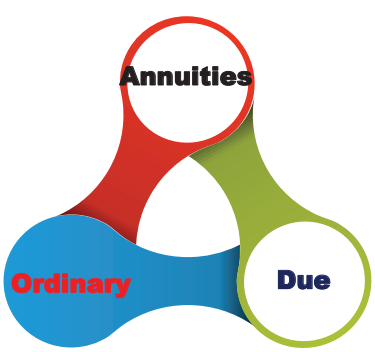
Annuities

An **annuity** is a stream consisting of a fixed number of equal cash flows paid at regular intervals. An annuity a stream of equal cash flows arriving at a regular interval and ending after a specified period. Therefore, the difference between an annuity and a perpetuity is that an annuity ends after some fixed number of payments whereas a perpetuity continues forever. Most car loans, mortgages, and some bonds are annuities. We represent the cash flows of an annuity on a time line as follows:



Meaning & Types of Annuities

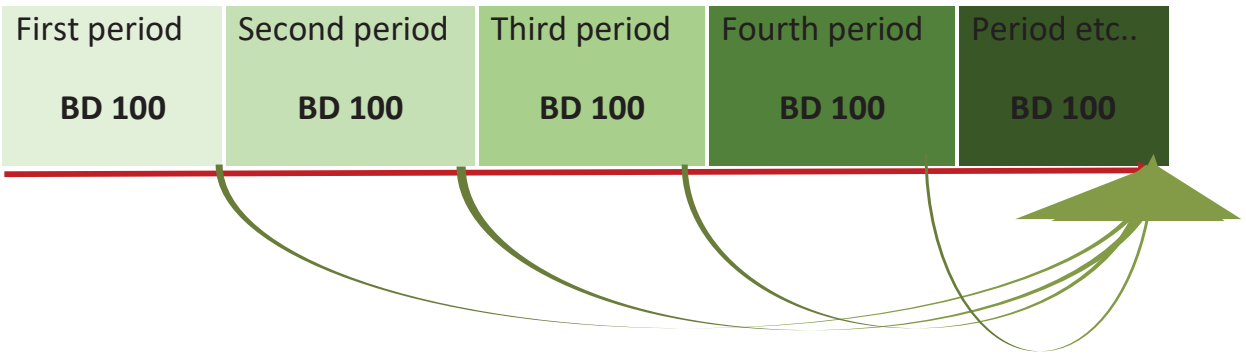
If there are several principals that is paid in a regular way at equal period of time this situation is called (annuities) and if it is paid by equal sums it is called (equal annuities) This payment could be paid monthly, quarterly, semi- annually or yearly. There are two types of annuities.



A- Payment / Ordinary Annuities:

2-1-1 Payment (Ordinary) Annuities.

- Is a payment paid at the end of each period (end of each month– end of each six months – end of each year).



Tips 2-1-1:

- In the previous unit, we dealt with the compounding of a single sum. Suppose that a specific amount is set aside each period (e.g, each year) and we want to know how much will it be in the account after several years.
- If one takes a loan and has to pay it after five years at equal annuities every three months, he will not pay anything when he receives the loan. The payment will start after three months and then he will pay the annuity at the end of every three-month period.

For Example: Suppose there are five annuities deposit of BD100 at the end of each year, with interest of 7% annually, starting a year from now. What will be the amount at the end of four years? A simple diagram will illustrate this.

The first BD100 will generate an amount for four years; the second amount will be for three years and so on. This problem could be solved by making separate calculations:

➢ BD100 Paid at the end of each year

0 1st year 1 2nd year 2 3rd year 3 4th year 4

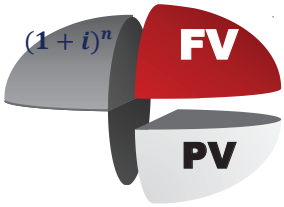
$FV=100(1+0.07)^3 = BD122.504$

$FV=100(1+0.07)^2 = BD114.490$

$FV=100(1+0.07)^1 = BD107.000$

$FV=100(1+0.07)^0 = BD100.000$

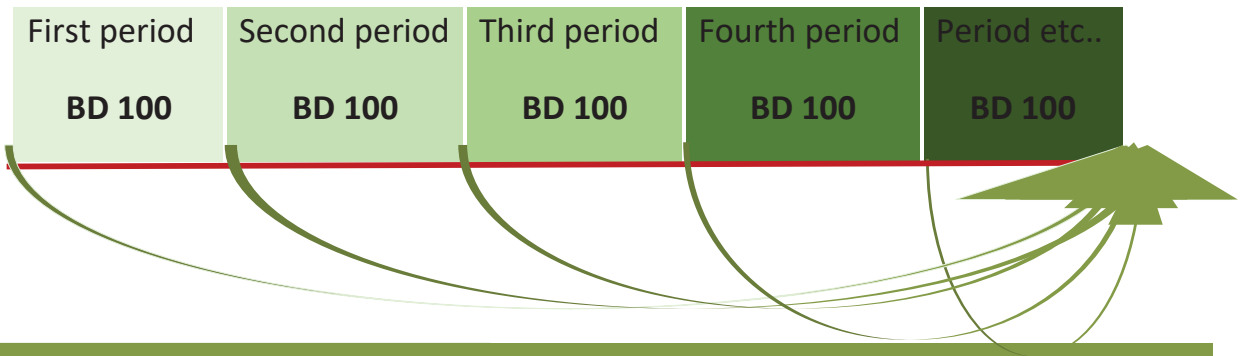
Total Sn = BD443.994



B- Investment / Due Annuities:

2-2-1 Investment (Due) Annuities.

- Is paid at the beginning of each period (begins every month – begins every six months – begins every year).



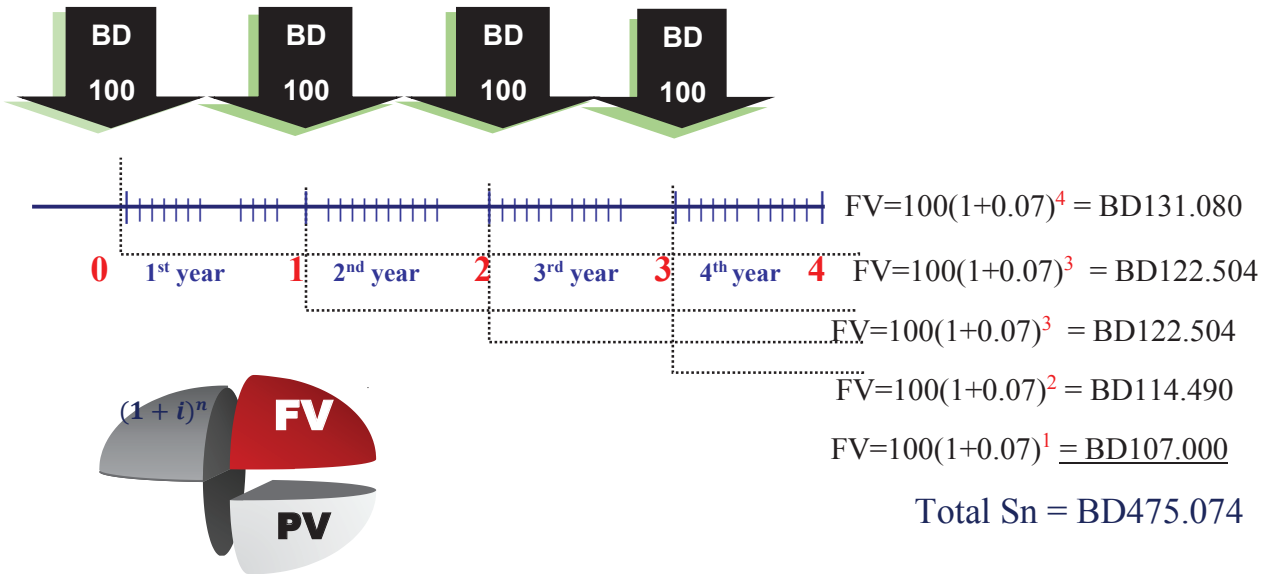
Tips 2-2-1:

- To finance college education of a just–born child, parents will need sum of money after 18 years from date of birth. The parents will have to deposit at the beginning of each year a sum of money. How much should they put aside each year?
- If a man signs a contract with an insurance company for life insurance, he should pay an annual annuity every year throughout 15 years. As soon as he signs the contract, he should pay the first annuity and he will pay the second annuity at the beginning of the second year, and so on.

For Example: Suppose four annuities' deposits of BD100 at the beginning of each year will be made to an account paying 7 % annually, starting a year from now. What will be the amount at the end of four years? A simple diagram will illustrate this.

The first BD100 will generate an amount for four years; the second amount will be for three years and so on. This problem could be solved by making separate calculations.

➢ Paid BD 100 at the beginning of each year



When you borrow money from a bank, you pay interest. Interest is really a fee charged for borrowing the money, it is a percentage charged on the principal amount for a period of time usually a year.

If you want to know how much interest you will earn on your investment or if you want to know how much you will pay above the cost of the principal amount on a loan or mortgage, you will need to understand how compound interest works.

Interest is computed on the principal sum plus the accrued interest. At the beginning of the new interest period, all interest is added to the principal, forming a new principal figure on which interest is then calculated. This process repeats itself as each interest period-interest may be compounded daily, monthly, semiannually or annually.

The key to understanding "compound interest" is to distinguish it from "simple interest". The following example illustrates the differences between simple interest and compound interest.

In this chapter, we will explain the importance of time in increasing the interest that the bank or customer gets when lending or depositing.

2-2-2 Investment (Due) Annuities.

- The ability to calculate the value of money at different points in time is one of the most important skills you will develop in finance. It is key to understanding the material ahead as well as making financial choices for your future. The time value of money (TVM) refers to a dollar in hand today being worth than a dollar received in the future. Today's dollar can be invested in an interest account that grows the value over time

2.3 Future and Present Value of Annuities

Future Value of Annuities

To compute the future value (compound amount) for one annuity monetary unit by using an electronic calculator or you can use the facilities, which are provided in the Microsoft office excel to compute the compound amount for one annuity monetary unit with a certain formula. You can easily appreciate the use of table No.3 when you consider how much time it saves. The calculation involves a lot of work and many chances to make mistakes if many periods are involved.

Present Value of Annuities

Up until this point, we have been moving money forward in time, we know how much we have to begin with and are trying to determine how much that sum will grow in a certain number of years when compounded at a specific rate.

We are now going to look at the reverse question: What is the value in today's Bahraini Dinar of a sum of money to be received in the future?
In this case, we are moving future money back to the present. We will be determining the **present value** of a lump sum, which in simple terms in the current value of a future payment. What we will be doing is, in fact, nothing other than inverse compounding. The differences in these techniques come about merely from the investor's point of view. In compounding, we explained earlier about the growth rate and investment in determining the present value, and we will explain the present value of money and equal payments that will be obtained or paid in the future.

Present Value of an annuity due is used to derive the current value of a series of cash payments that are expected to be made on predetermined future dates and in predetermined amounts. The calculation is usually made to decide if you should take a lump sum payment now, or to instead receive a series of cash payments in the future.

A. Ordinary Annuities

An ordinary annuity is a series of equal payments made at the end of consecutive periods over a fixed length of time. While the payments in an ordinary annuity can be made as frequently as every week, in practice they are generally made monthly, quarterly, semi-annually, or annually.

Tips 2-3-1: Formulas of Future value for Ordinary Annuities

- If you are required to calculate, the **future and present value** of the two type's annuities to ease the effort involved. The next formula will be use to calculate this.

Title	Method	Ordinary
Future Value	Table	$FV_n = PMT \times FVIF\ n,i$
	Calculator	$FV_n = PMT \times [\frac{(1+i)^n - 1}{i}]$
Present Value	Table	$PV_n = PMT \times PVIF\ n,i$
	Calculator	$PV_n = PMT \times [\frac{1-(1+i)^{-n}}{i}]$

2-3-1

- FV_n : Future value of ordinary annuity.
- PV_n : Present value of ordinary annuity.
- PMT : Equal Payments.
- n : Number of Annuities.
- i : Interest Rate.
- $FVIF\ n,i$: Searching from future value of ordinary annuity table.
- $PVIF\ n,i$: Searching from present value of ordinary annuity table.

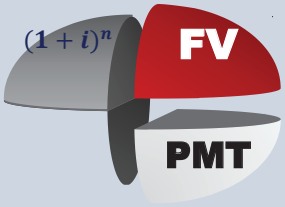
Example 2-3-1:

A trader paid an annuity of BD150 at the end of each year for 3 years at an interest rate of 5% annually. Find the following:

1- Future value (amount) and interest at the end of the period.

2- Present value (amount) at the end of the period.

Answer:



$$1\text{- } FV_n = PMT \times \left[\frac{(1+i)^n - 1}{i}\right]$$

$$= 150 \times \left[\frac{(1+5\%)^3 - 1}{5\%}\right]$$

$$= 150 \times \left[\frac{0.157625}{0.05}\right]$$

$$= 150 \times [3.1525] = \text{BD } 472.875$$

OR by using interest table $= 150 \times 3.1525 = \text{BD } 472.875$

TABLE (FV of Ordinary Annuity) (annuity in arrears … end of period)

n \ i	4.00%	5.00%	6.00%	7.00%	8.00%	9.00%	10.00%	11.00%	12.00%
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	2.04000	2.05000	2.06000	2.07000	2.08000	2.09000	2.10000	2.11000	2.12000
3	3.12160	3.15250	3.18360	3.21490	3.24640	3.27810	3.31000	3.34210	3.37440

To calculate compounded Interest:

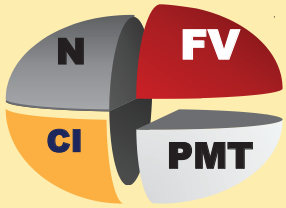
Important Points 2-3-1: Calculate Compound Interest

$$CI = FV_n - (PMT \times n)$$

$$= 472.875 - (150 \times 3)$$

$$= 472.875 - 450$$

$$= \text{BD } 22.875$$



Example 2-3-1:

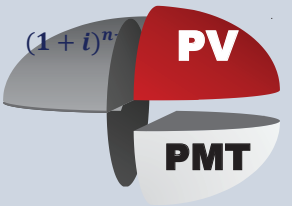
Answer:

2- $PV_n = PMT \times \left[\frac{1-(1+i)^{-n}}{i}\right]$

$= 150 \times \left[\frac{1-(1+5\%)^{-3}}{5\%}\right]$

$= 150 \times \left[\frac{0.1362}{0.05}\right]$

$= 150 \times 2.72325 = \text{BD } 408.488$



OR by using interest table $= 150 \times 2.72325 = \text{BD } 408.488$

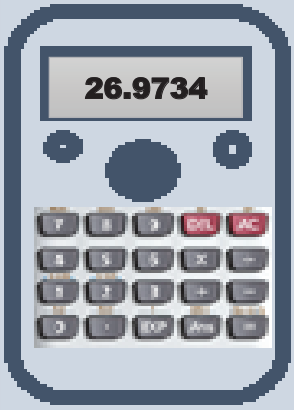
TABLE (PV of Ordinary Annuity) **(annuity in arrears … end of period)**

n \ i	5.00%	6.00%	7.00%	8.00%	9.00%	10.00%	11.00%	12.00%
1	0.95238	0.94340	0.93458	0.92593	0.91743	0.90909	0.90090	0.89286
2	1.85941	1.83339	1.80802	1.78326	1.75911	1.73554	1.71252	1.69005
3	2.72325	2.67301	2.62432	2.57710	2.53129	2.48685	2.44371	2.40183
4	3.54595	3.46511	3.38721	3.31213	3.23972	3.16987	3.10245	3.03735

Example 2-3-1:

- Calculate the future value and interest of an ordinary annuity of BD200 paid 4 times a year for 6 years if the nominal rate is 4% annually.

Answer:



No. of annuities (n) = 6 × 4 = 24

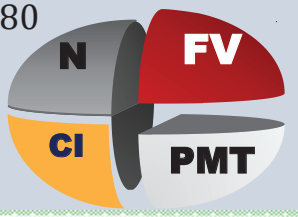
Partial rate (i) = 4% ÷ 4 = 1%

$$1-\ FV_n = PMT \times \left[\frac{(1+i)^n-1}{i}\right]$$

$$= 200 \times \left[\frac{(1+1\%)^{24}-1}{1\%}\right]$$

= 200 x 26.9734 = BD 5394.680

OR by using interest table = 200 x 26.9734 = BD 5394.680



2- CI = FV_n - (PMT x n)

= 5394.680 - (200 x 24) = BD 594.680

TABLE (FV of Ordinary Annuity)
(annuity in arrears ... end of period)

n \ i	0.25%	0.50%	0.75%	1.00%	1.50%	2.00%	2.50%	3.00%
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	2.00250	2.00500	2.00750	2.01000	2.01500	2.02000	2.02500	2.03000
3	3.00751	3.01502	3.02256	3.03010	3.04522	3.06040	3.07563	3.09090
4	4.01503	4.03010	4.04523	4.06040	4.09090	4.12161	4.15252	4.18363
5	5.02506	5.05025	5.07556	5.10101	5.15227	5.20404	5.25633	5.30914
6	6.03763	6.07550	6.11363	6.15202	6.22955	6.30812	6.38774	6.46841
7	7.05272	7.10588	7.15948	7.21354	7.32299	7.43428	7.54743	7.66246
8	8.07035	8.14141	8.21318	8.28567	8.43284	8.58297	8.73612	8.89234
9	9.09053	9.18212	9.27478	9.36853	9.55933	9.75463	9.95452	10.15911
10	10.11325	10.22803	10.34434	10.46221	10.70272	10.94972	11.20338	11.46388
11	11.13854	11.27917	11.42192	11.56683	11.86326	12.16872	12.48347	12.80780
12	12.16638	12.33556	12.50759	12.68250	13.04121	13.41209	13.79555	14.19203
13	13.19680	13.39724	13.60139	13.80933	14.23683	14.68033	15.14044	15.61779
14	14.22979	14.46423	14.70340	14.94742	15.45038	15.97394	16.51895	17.08632
15	15.26537	15.53655	15.81368	16.09690	16.68214	17.29342	17.93193	18.59891
16	16.30353	16.61423	16.93228	17.25786	17.93237	18.63929	19.38022	20.15688
17	17.34429	17.69730	18.05927	18.43044	19.20136	20.01207	20.86473	21.76159
18	18.38765	18.78579	19.19472	19.61475	20.48938	21.41231	22.38635	23.41444
19	19.43362	19.87972	20.33868	20.81090	21.79672	22.84056	23.94601	25.11687
20	20.48220	20.97912	21.49122	22.01900	23.12367	24.29737	25.54466	26.87037
21	21.53341	22.08401	22.65240	23.23919	24.47052	25.78332	27.18327	28.67649
22	22.58724	23.19443	23.82230	24.47159	25.83758	27.29898	28.86286	30.53678
23	23.64371	24.31040	25.00096	25.71630	27.22514	28.84496	30.58443	32.45288
24	24.70282	25.43196	26.18847	26.97346	28.63352	30.42186	32.34904	34.42647

Example 2-3-2:

- Calculate the present value of an ordinary annuity of BD200 paid 4 times a year for 6 years if the nominal rate is 4% annually.

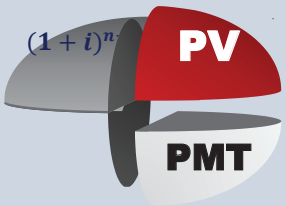
Answer:

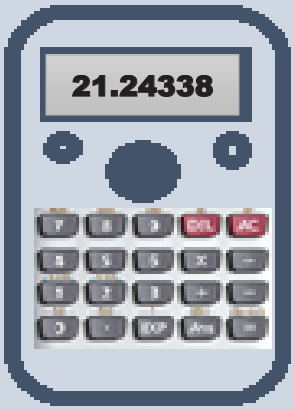
$$PV_n = PMT \times \left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$= 200 \times \left[\frac{1-(1+1\%)^{-24}}{1\%}\right]$$

$$= 200 \times \left[\frac{1-0.787566}{0.01}\right]$$

$$= 200 \times 21.24339 = \text{BD } 4248.678$$





OR by using interest table = 200 x 21.24339 = BD 4248.678

TABLE (PV of Ordinary Annuity)
(annuity in arrears ... end of period)

n \ i	0.25%	0.50%	0.75%	1.00%	1.50%	2.00%	2.50%	3.00%	4.00%
1	0.99751	0.99502	0.99256	0.99010	0.98522	0.98039	0.97561	0.97087	0.96154
2	1.99252	1.98510	1.97772	1.97040	1.95588	1.94156	1.92742	1.91347	1.88609
3	2.98506	2.97025	2.95556	2.94099	2.91220	2.88383	2.85602	2.82861	2.77509
4	3.97512	3.95050	3.92611	3.90197	3.85438	3.80773	3.76197	3.71710	3.62990
5	4.96272	4.92587	4.88944	4.85343	4.78264	4.71346	4.64583	4.57971	4.45182
6	5.94785	5.89638	5.84560	5.79548	5.69719	5.60143	5.50813	5.41719	2.24214
7	6.93052	6.86207	6.79764	6.72819	6.59821	6.47199	6.34939	6.23028	6.00205
8	7.91074	7.82296	7.73661	7.65168	7.48593	7.32548	7.17014	7.01969	6.73274
9	8.88852	8.77906	8.67158	8.56602	8.36052	8.16224	7.97087	7.78611	7.43533
10	9.86386	9.73041	9.59958	9.47130	9.22218	8.98259	8.75206	8.53020	8.11090
11	10.83677	10.67703	10.52067	10.36763	10.07112	9.78685	9.51421	9.25262	8.76048
12	11.80725	11.61893	11.43491	11.25505	10.90751	10.57534	10.25776	9.95400	9.38507
13	12.27532	12.55615	12.34235	12.13374	11.73153	11.34837	10.98318	10.63496	9.98565
14	13.74096	13.48871	13.24302	13.00370	12.54338	12.10625	11.69091	11.29607	10.56312
15	14.66504	14.41662	14.13699	13.86505	13.34323	12.84926	12.38138	11.93794	11.11839
16	15.66504	15.33993	15.02431	14.71787	14.13126	13.57771	13.05500	12.56110	11.65230
17	16.62348	16.25863	15.90502	15.56225	14.90765	14.29187	13.71220	13.16612	12.16567
18	17.57953	17.17277	16.77918	16.39827	15.67256	14.99203	14.35336	13.75351	12.65930
19	18.53320	18.08236	17.64683	17.22601	16.42617	15.67846	14.97889	14.32380	13.13394
20	19.48449	18.98742	18.50802	18.04555	17.16864	16.35143	15.58916	14.87747	13.59033
21	20.43340	19.88798	19.36280	18.85698	17.90014	17.01121	16.18455	15.41502	14.02916
22	21.37995	20.78406	20.21121	19.66038	18.26802	17.65805	16.76541	15.93692	14.45112
23	22.32414	21.67568	21.05231	20.45582	19.33086	18.29220	17.33211	16.44361	14.85684
24	23.26598	22.56287	21.88915	21.24339	20.03041	18.91393	17.88499	16.93554	15.24696

B. Investment (Due) Annuities

An annuity due, in which payments are made at the beginning of each period are required to be made at the start of each annuity period instead of the end of the period. The payments are generally fixed and there are two values for an annuity, one would be future value, and another would be present value

Tips 2-3-2: Formulas of Future value for Investment Annuities

- If you are required to calculate, the **future and present value** of the two type's annuities to ease the effort involved. The next formula will be used to calculate this.

Title	Method	Annuity due
Future Value	Table	$FV_{nd} = PMT \times FVIF_d\ n,i$
	Calculator	$FV_{nd} = PMT \times [\frac{(1+i)^n-1}{i}] \times (1+i)$
Present Value	Table	$PV_{nd} = PMT \times PVIF_d\ n,i$
	Calculator	$PV_{nd} = PMT \times [\frac{1-(1+i)^{-n}}{i}] \times (1+i)$

2-3-2

- FV_{nd} : Future value of annuity due.
- PV_{nd} : Present value of annuity due.
- PMT : Equal Payments.
- n : Number of Annuities.
- i : Interest Rate.
- $FVIF_d$ n ,i: Searching from future value of annuity due table.
- $PVIF_d$ n ,i: Searching from present value of annuity due table.

Example 2-3-3:

A trader paid an annuity of BD120 at the beginning of each six months at an interest rate of 8% annually. Find the following:

1-Future value (amount) and interest at the end of 4 years.

2- Present value of the annuities at the end of the period.

Answer:

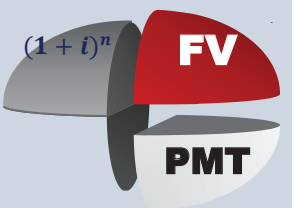
No. of annuities (n) = $4 \times 2 = 8$

Partial rate (i) $= 8\% \div 2 = 4\%$

1- $FV_{nd} = PMT \times [\frac{(1+i)^n - 1}{i}] \times (1+i)$

$= 120 \times [\frac{(1+4\%)^8 - 1}{4\%}] \times (1+4\%)$

$= 120 \times 9.21423 \times 1.04 = BD\ 1149.936$



OR by using interest table $= 120 \times 9.5828 = BD\ 1149.936$

TABLE (FV of Annuity Due)

(annuity in advance – beginning of period payment)

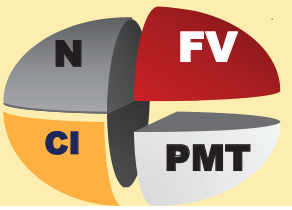
n \ i	4.00%	5.00%	6.00%	7.00%	8.00%	9.00%	10.00%	%11.00	12.00%
1	1.04000	1.05000	1.06000	1.07000	1.08000	1.09000	1.10000	1.11000	1.12000
2	2.12160	2.15250	2.18360	2.21490	2.24640	2.27810	2.31000	2.34210	2.37440
3	3.24646	3.31013	3.37462	3.43994	3.50611	3.7313	3.64100	3.70973	3.77933
4	4.41632	4.52563	4.63709	4.75074	4.86660	4.98471	5.10510	5.22780	5.35285
5	5.63298	5.80191	5.97532	6.15329	6.33593	6.52333	6.71561	6.91286	7.11519
6	6.89829	7.14201	7.39384	7.65402	7.92282	8.20043	8.48717	8.78327	9.08901
7	8.21423	8.54911	8.89747	9.25980	9.63663	10.02847	10.43589	10.85943	11.29969
8	9.58280	10.02656	10.49132	10.97799	11.48456	12.02104	12.57948	13.16397	13.77566
9	11.00611	11.57789	12.18079	12.81645	13.48656	14.19293	14.93742	15.72201	16.54874

Important Points 2-3-1:

$CI = FV_{nd} - (PMT \times n)$

$= 1149.936 - (120 \times 8)$

$= 1149.936 - 960 = BD\ 189.936$

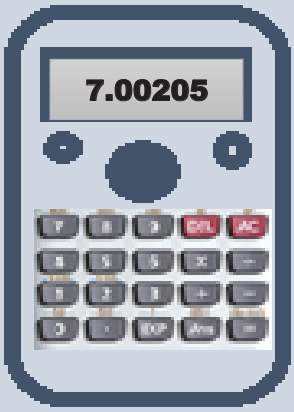


Example 2-3-3:

A trader paid an annuity of BD120 at the beginning of each six months at an interest rate of 8% annually. Find the following:

1-Future value (amount) and interest at the end of 4 years.

2- Present value of the annuities at the end of the period



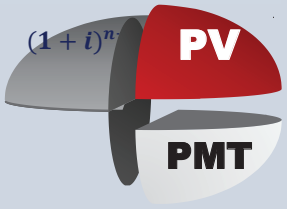
Answer:

$$\text{2- } PV_{nd} = PMT \times \left[\frac{1-(1+i)^{-n}}{i}\right] \times (1+i)$$

$$= 120 \times \left[\frac{1-(1+4\%)^{-8}}{4\%}\right] \times (1+4\%)$$

$$= 120 \times \left[\frac{1-0.730690}{0.04}\right] \times (1+4\%)$$

$$= 120 \times 6.732744 \times 1.04 = \text{BD } 840.246$$



OR by using interest table $= 120 \times 7.00205 =$ BD 840.246

TABLE (PV of Annuity Due)

(annuity in advance … beginning of period)

n \ i	0.25%	0.50%	0.75%	1.00%	1.50%	2.00%	2.50%	3.00%	4.00%
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	1.99751	1.99502	1.99256	1.99010	1.98522	1.98039	1.97561	1.97087	1.96154
3	2.99252	2.98510	2.97772	2.97040	2.95588	2.94156	2.92742	2.91347	2.88609
4	3.98506	3.97025	3.95556	3.94099	3.912200	3.88388	3.85602	3.82861	3.77509
5	4.97512	4.95050	4.92611	4.90197	4.85438	4.80773	4.76197	4.71710	4.62990
6	5.96272	5.92587	5.88944	5.85343	5.78264	5.71346	5.64583	5.57971	5.45182
7	6.94785	6.89638	6.84560	6.79548	6.69719	6.60143	6.50813	6.41719	6.24214
8	7.93052	7.86207	7.79464	7.72819	7.59821	7.47199	7.34939	7.23028	7.00205

Example 2-3-4:

Calculate the future value and the present value of annuities due of BD80 paid each 4 months a year for 3year and 8 months if the nominal rate is 2.5% thirdly.

Answer:

No. of annuities (n) = 3.667 × 3= 11

Partial rate (i) = 2.5%

1- $FV_{nd} = PMT \times [\frac{(1+i)^n - 1}{i}] \times (1+i)$

$= 80 \times [\frac{(1+2.5\%)^{11} - 1}{2.5\%}] \times (1.025)$

$= 80 \times 12.48347 \times (1.025) = BD\ 1023.645$

2- $PV_{nd} = PMT \times [\frac{1-(1+i)^{-n}}{i}] \times (1+i)$

$= 80 \times [\frac{1-(1+2.5\%)^{-11}}{2.5\%}] \times (1+2.5\%)$

$= 80 \times [\frac{1-0.76214}{0.025}] \times 1.025$

$= 80 \times 9.5142 \times 1.025 = BD\ 780.164$

TABLE (FV of Ordinary Annuity)

(annuity in arrears … end of period)

n \ i	0.25%	0.50%	0.75%	1.00%	1.50%	2.00%	2.50%	3.00%
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	2.00250	2.00500	2.00750	2.01000	2.01500	2.02000	2.02500	2.03000
3	3.00751	3.01502	3.02256	3.03010	3.04522	3.06040	3.07563	3.09090
4	4.01503	4.03010	4.04523	4.06040	4.09090	4.12161	4.15252	4.18363
5	5.02506	5.05025	5.07556	5.10101	5.15227	5.20404	5.25633	5.30914
6	6.03763	6.07550	6.11363	6.15202	6.22955	6.30812	6.38774	6.46841
7	7.05272	7.10588	7.15948	7.21354	7.32299	7.43428	7.54743	7.66246
8	8.07035	8.14141	8.21318	8.28567	8.43284	8.58297	8.73612	8.89234
9	9.09053	9.18212	9.27478	9.36853	9.55933	9.75463	9.95452	10.15911
10	10.11325	10.22803	10.34434	10.46221	10.70272	10.94972	11.20338	11.46388
11	11.13854	11.27917	11.42192	11.56683	11.86326	12.16872	12.48347	12.80780

2.4 Finding the Value of Annuity

A. Finding Value of Ordinary Annuities

Important Points 2-4-1:

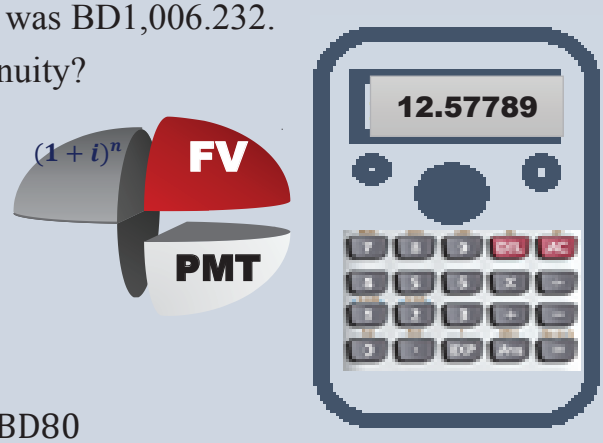
$$PMT = FV_n \div \left[\frac{(1+i)^n - 1}{i}\right]$$

OR

$$PMT = PVn \div \left[\frac{1-(1+i)^{-n}}{i}\right]$$

Example 2-4-1:

Amal pays a loan installment at the end of each year for 10 years at 5 % annually. If her balance at the end of the period was BD1,006.232. What is the value of each ordinary annuity?



Answer:

$$PMT = FV_n \div \left[\frac{(1+i)^n - 1}{i}\right]$$

$$= 1006.232 \div \left[\frac{(1+5\%)^{10} - 1}{5\%}\right]$$

$$= 1006.232 \div \left[\frac{1.6289 - 1}{0.05}\right]$$

$$= 1006.232 \div 12.57789 = \text{BD}80$$

TABLE (FV of Ordinary Annuity)
(annuity in arrears ... end of period)

n \ i	4.00%	5.00%	6.00%	7.00%	8.00%	9.00%	10.00%	11.00%	12.00%
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	2.04000	2.05000	2.06000	2.07000	2.08000	2.09000	2.10000	2.11000	2.12000
3	3.12160	3.15250	3.18360	3.21490	3.24640	3.27810	3.31000	3.34210	3.37440
4	4.24646	4.31013	4.37462	4.43994	4.50611	4.57313	4.64100	4.70973	4.77933
5	5.41632	5.52563	5.63709	5.75074	5.86660	5.98471	6.10510	6.22780	6.35285
6	6.63298	6.80191	6.97532	7.15329	7.33593	7.52333	7.71561	7.91286	8.11519
7	7.89829	8.14201	8.39384	8.65402	8.92280	9.20043	9.48717	9.78327	10.08901
8	9.21423	9.54911	9.89747	10.25980	10.63663	11.02847	11.43589	11.85943	12.29969
9	10.58280	11.02656	11.49132	11.97799	12.48756	13.02104	13.57948	14.16397	14.77566
10	12.00611	12.57789	13.18079	13.81645	14.48656	15.19293	15.93742	16.72201	17.54874

Example 2-4-2:

Abdulla deposited at the end of each four months for 5 years at 12 % annually compounded interest thirdly. If the present value of his money now is BD5559.195 - What is the value of each annuity?

Answer:

$i = 12\% \div 3 = 4\%$ $\quad n = 5 \times 3 = 15$

$$PMT = PVn \div \left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$= 5559.195 \div \left[\frac{1-(1+4\%)^{-15}}{4\%}\right]$$

$$= 5559.195 \div 11.11839 = \text{BD } 500$$

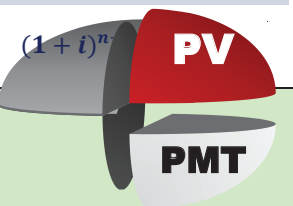


TABLE (PV of Ordinary Annuity)

(annuity in arrears ... end of period)

n \ i	0.25%	0.50%	0.75%	1.00%	1.50%	2.00%	2.50%	3.00%	4.00%
1	0.99751	0.99502	0.99256	0.99010	0.98522	0.98039	0.97561	0.97087	0.96154
2	1.99252	1.98510	1.97772	1.97040	1.95588	1.94156	1.92742	1.91347	1.88609
3	2.98506	2.97025	2.95556	2.94099	2.91220	2.88383	2.85602	2.82861	2.77509
4	3.97512	3.95050	3.92611	3.90197	3.85438	3.80773	3.76197	3.71710	3.62990
5	4.96272	4.92587	4.88944	4.85343	4.78264	4.71346	4.64583	4.57971	4.45182
6	5.94785	5.89638	5.84560	5.79548	5.69719	5.60143	5.50813	5.41719	2.24214
7	6.93052	6.86207	6.79764	6.72819	6.59821	6.47199	6.34939	6.23028	6.00205
8	7.91074	7.82296	7.73661	7.65168	7.48593	7.32548	7.17014	7.01969	6.73274
9	8.88852	8.77906	8.67158	8.56602	8.36052	8.16224	7.97087	7.78611	7.43533
10	9.86386	9.73041	9.59958	9.47130	9.22218	8.98259	8.75206	8.53020	8.11090
11	10.83677	10.67703	10.52067	10.36763	10.07112	9.78685	9.51421	9.25262	8.76048
12	11.80725	11.61893	11.43491	11.25505	10.90751	10.57534	10.25776	9.95400	9.38507
13	12.27532	12.55615	12.34235	12.13374	11.73153	11.34837	10.98318	10.63496	9.98565
14	13.74096	13.48871	13.24302	13.00370	12.54338	12.10625	11.69091	11.29607	10.56312
15	14.66504	14.41662	14.13699	13.86505	13.34323	12.84926	12.38138	11.93794	11.11839

B. Finding Value of Annuities Due

Important Points 2-4-2:

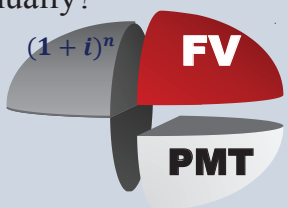
$$PMT = FV_{nd} \div \left[\frac{(1+i)^n - 1}{i}\right] \div (1+i)$$

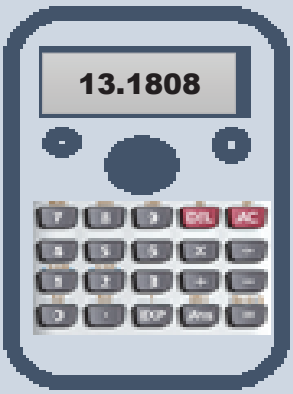
OR

$$PMT = PV_{nd} \div \left[\frac{1-(1+i)^{-n}}{i}\right] \div (1+i)$$

Example 2-4-3:

What is annually annuity paid at the beginning of each year, if the future value is BD2794.329 in 10 years at 6 % annually?





Answer:

$i = 6\%$ $n = 10$

$$PMT = FV_{nd} \div \left[\frac{(1+i)^n - 1}{i}\right] \div (1+i)$$

$$= 2794.329 \div \left[\frac{(1+6\%)^{10} - 1}{6\%}\right] \div (1+6\%)$$

$PMT = 2794.329 \div 13.1808 \div 1.06 = BD\ 200$

OR by using interest table $= 2794.329 \div 13.97164 = BD\ 200$

TABLE (FV of Annuity Due)

(annuity in advance – beginning of period payment)

n \ i	4.00%	5.00%	6.00%	7.00%	8.00%	9.00%	10.00%	%11.00	12.00%
1	1.04000	1.05000	1.06000	1.07000	1.08000	1.09000	1.10000	1.11000	1.12000
2	2.12160	2.15250	2.18360	2.21490	2.24640	2.27810	2.31000	2.34210	2.37440
3	3.24646	3.31013	3.37462	3.43994	3.50611	3.7313	3.64100	3.70973	3.77933
4	4.41632	4.52563	4.63709	4.75074	4.86660	4.98471	5.10510	5.22780	5.35285
5	5.63298	5.80191	5.97532	6.15329	6.33593	6.52333	6.71561	6.91286	7.11519
6	6.89829	7.14201	7.39384	7.65402	7.92282	8.20043	8.48717	8.78327	9.08901
7	8.21423	8.54911	8.89747	9.25980	9.63663	10.02847	10.43589	10.85943	11.29969
8	9.58280	10.02656	10.49132	10.97799	11.48456	12.02104	12.57948	13.16397	13.77566
9	11.00611	11.57789	12.18079	12.81645	13.48656	14.19293	14.93742	15.72201	16.54874
10	12.48635	13.20679	13.97164	14.78360	15.64549	16.56029	17.53117	18.56143	19.65458

Exercises:

1- Ahmed paid an annuity of BD400 at the end of each year for 7 years at an interest rate of 3% annually. Find the following:

a- Future value (amount) and interest at the end of the period.

b- Present value (amount) at the end of the period.

2- A trader paid an annuity of BD600 at the beginning of each three months at an interest rate of 6% annually. Find the following:

a- Future value (amount) and interest at the end of 10 years.

b- Present value of the annuities at the end of the period

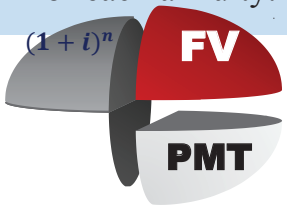
3- Calculate the future value and interest of an ordinary annuity of BD800 paid 4 times a year for 6 years if the nominal rate is 4% annually.

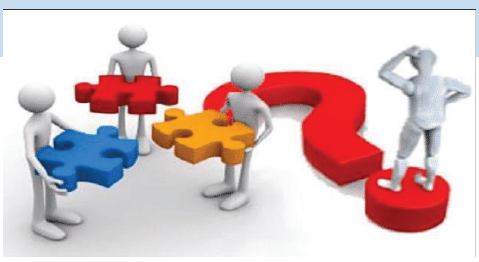
4- At the beginning of every 4 months, Nasser deposited an annuity in a bank for 7 years at 9% annually. If the accumulated fund for him became $ 3249.048. Find how much Nasser deposited every 4 months.

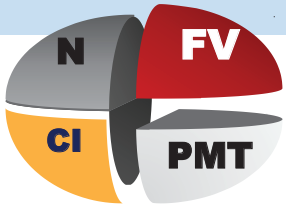
5- Rayan deposited at the middle and at the end of each year an equal payment for 10 years at 5% annually. If the total amount of annuities at the end of the period was BD5236.664. Find the value of each annuity.

6- What semi – annually payment will accumulate to BD1080.549 in five years at 8% annually compounded semi- annually?

7- Bilal paid an annuity at the end of every year at 6% annually if the amount of annuities after 10 years was BD922.656. Find the value of each annuity.







2.5 Loan Amortization Schedule

Loan Payment Methods

When you borrow money, there are three different ways to repay the loan:

1. You can pay off the **principal** (the original loan amount that you borrowed) and all the **interest (**the amount the lender charges you for borrowing money) at one time at the maturity date of the loan. This kind of loan is called a **discount loan.**

2. You can make interest payments as you go and then pay the principal and final interest payment at the maturity date. This kind of loan is called **interest-only loan**.

3. You can pay both principal and interest as you go by making equal payments each period. This kind of loan is called **amortized loan.**

Amortized Loan

The most common way for consumers to pay off a loan is to make equal payments each period, with a portion going to the interest for the period and reminder applied against the outstanding principal. This common payment method is an application of an annuity presented earlier in this chapter.

This problem is a simple application of the present value payment equation:

Important Points 2-5-1:

$$PMT = PVn \div \left[\frac{1-(1+i)^{-n}}{i}\right]$$

OR $$PMT = \left[\frac{PV_n}{(PVIF\ n,i)}\right]$$

Example 2-5-1:

A trader borrowed BD50,548.320 from a bank at compound interest rate 6% annually with 5 equal annual payments.

REQUIRED:

a- Compute annual payments.

b- Prepare amortization loan schedule

Answer:

$$PMT = 50{,}548.320 \div \left[\frac{1-(1+.06)^{-5}}{0.06}\right]$$

$$= 50{,}548.320 \div \left[\frac{1-0.7473}{0.06}\right]$$

$$= 50{,}548.320 \div 4.21236$$

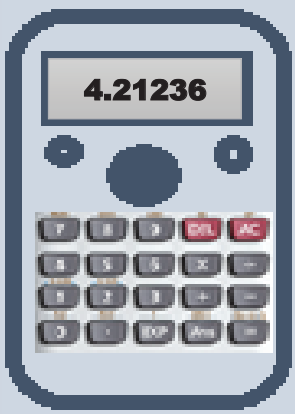
$$= BD12000$$

OR: $$PMT = \left[\frac{PV_n}{(PVIF\ n,i)}\right]$$

$$= \left[\frac{50548.320}{(PVIF\ 5,6\%)}\right]$$

$$= \left[\frac{50548.320}{4.21236}\right]$$

$$= BD12{,}000$$



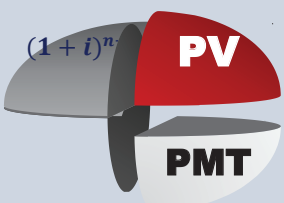


TABLE (PV of Ordinary Annuity)

(annuity in arrears … end of period)

n \ i	5.00%	6.00%	7.00%	8.00%	9.00%	10.00%	11.00%	12.00%
1	0.95238	0.94340	0.93458	0.92593	0.91743	0.90909	0.90090	0.89286
2	1.85941	1.83339	1.80802	1.78326	1.75911	1.73554	1.71252	1.69005
3	2.72325	2.67301	2.62432	2.57710	2.53129	2.48685	2.44371	2.40183
4	3.54595	3.46511	3.38721	3.31213	3.23972	3.16987	3.10245	3.03735
5	4.32948	4.21236	4.10020	3.99271	3.88965	3.79079	3.69590	3.60478

Example 2-5-1:

Steps to prepare loan amortization schedule

Year 1:

1- Simple Interest (SI) = 50,548.320 ×6%×1 =BD3032.899

2- Principal Reduction = PMT – SI
= 12000 – 3032.899 =BD8967.101

3- Remaining Principal = Beginning Year Principal - Principal Reduction
= 50,548.320 - 8967.101 =BD41581.219

Year 2:

1- Remaining Principal year1 = Beginning Principal Year2 = BD41581.219

2- Simple Interest (SI) = 41581.219×6%×1 =BD2494.873

3- Principal Reduction = PMT – SI
= 12000 – 2494.873=BD9505.127

4- Remaining Principal = Beginning Year Principal - Principal Reduction
= 41581.219- 9505.127=BD32076.092

Year 3:

1- Remaining Principal year1 = Beginning Principal Year2 = BD32076.092

2- Simple Interest (SI) = 32076.092×6%×1 =BD1924.566

3- Principal Reduction = PMT – SI
= 12000 – 1924.566=BD10075.434

4- Remaining Principal = Beginning Year Principal - Principal Reduction
= 32076.092- 10075.434=BD22000.658

Year 4:

1- Remaining Principal year1 = Beginning Principal Year2 = BD22000.658

2- Simple Interest (SI) = 22000.658×6%×1 =BD1320.039

3- Principal Reduction = PMT – SI
= 12000 – 1320.039=BD10679.961

4- Remaining Principal = Beginning Year Principal - Principal Reduction
= 22000.658- 10679.961=BD11320.697

Year 5:

1- Remaining Principal year1 = Beginning Principal Year2 = BD11320.697

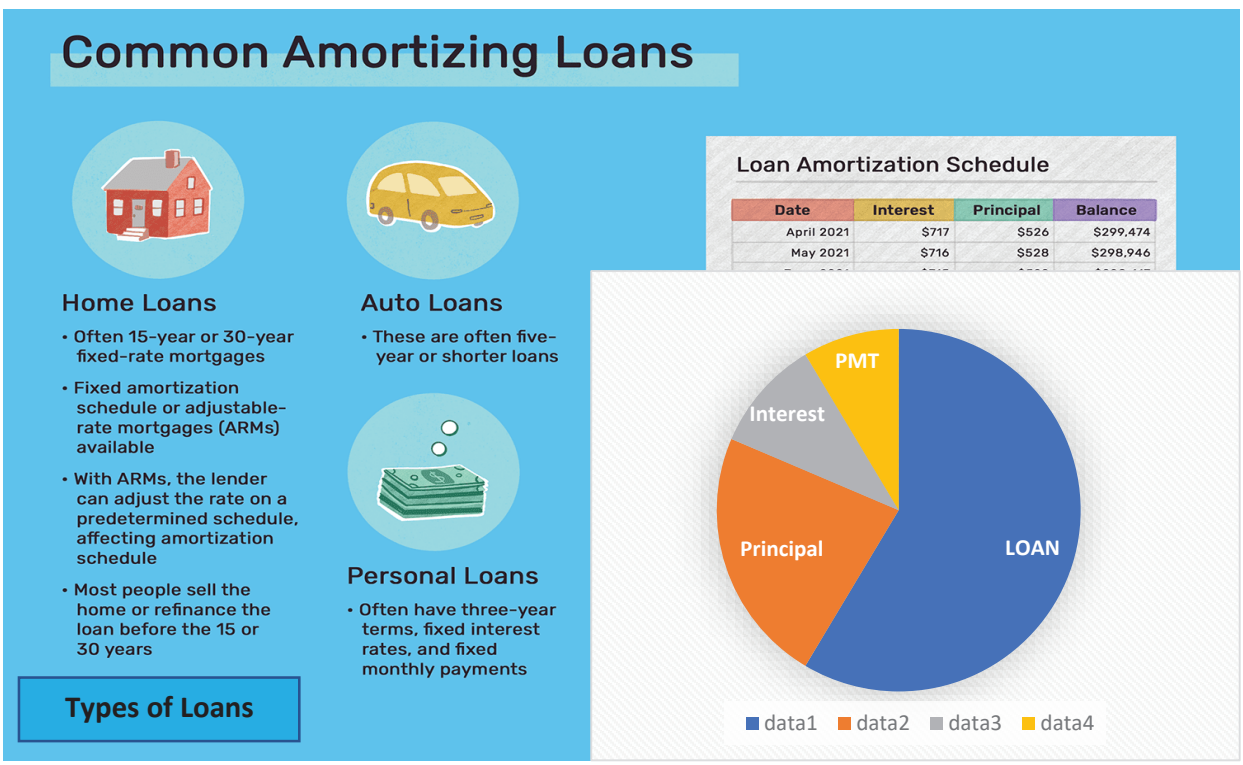
2- Simple Interest (SI) = 11320.697×6%×1 =BD679.242*

3- Principal Reduction = PMT – SI
= 12000 – 679.242=BD11320.697*

4- Remaining Principal = Beginning Year Principal - Principal Reduction
= 11320.697- 11320.697=BD0

Loan Amortization Schedule

year	Beginning Principal 1	Annual Payment 2	Interest Expense 3 = 1×6%	Principal Reduction 4 = 2 - 3	Remaining Principal 5 = 1 - 4
1	50,548.320	12,000	3,032.899	8967.101	41,581.219
2	41,581.219	12,000	2,494.873	9505.127	32,076.092
3	32,076.092	12,000	1,924.566	10075.434	22,000.658
4	22,000.658	12,000	1,320.039	10679.961	11,320.697
5	11,320.697	12,000	679.303*	11,320.697	0
Total	-	60,000	9451.680	50,548.320	-



Example 2-5-2:

Ahmed borrowed BD97,368 from a bank at compound interest rate 10% annually for 7 years.

REQUIRED:

a- Compute annual payments.

b- Prepare amortization loan schedule

Answer:

$$PMT = 97368 \div \left[\frac{1-(1+.10)^{-7}}{0.10}\right]$$

$$= 97368 \div \left[\frac{1-0.5132}{0.10}\right]$$

$$= 97368 \div 4.8684$$

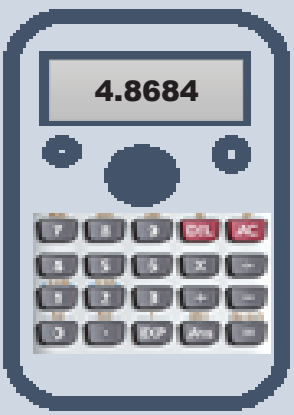
$$= BD20{,}000$$

OR: $$PMT = \left[\frac{PV_n}{(PVIF\ n, i)}\right]$$

$$= \left[\frac{97{,}368}{(PVIF\ 7, 10\%)}\right]$$

$$= \left[\frac{97{,}368}{4.8684}\right]$$

$$= BD20{,}000$$



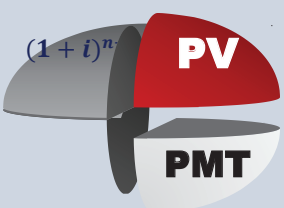
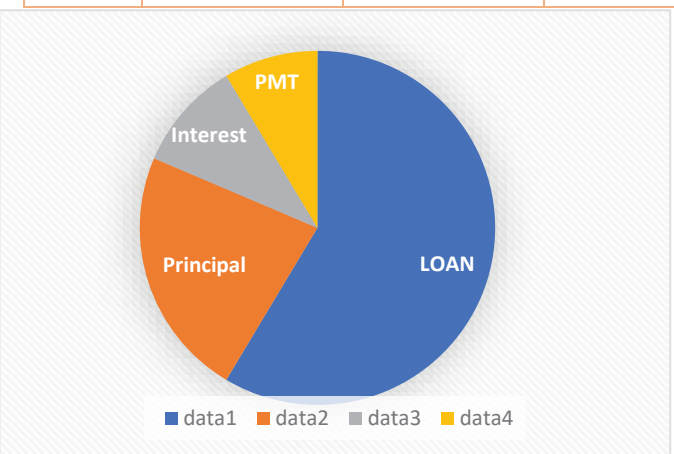


TABLE (PV of Ordinary Annuity)

(annuity in arrears … end of period)

n \ i	5.00%	6.00%	7.00%	8.00%	9.00%	10.00%	11.00%	12.00%
1	0.95238	0.94340	0.93458	0.92593	0.91743	0.90909	0.90090	0.89286
2	1.85941	1.83339	1.80802	1.78326	1.75911	1.73554	1.71252	1.69005
3	2.72325	2.67301	2.62432	2.57710	2.53129	2.48685	2.44371	2.40183
4	3.54595	3.46511	3.38721	3.31213	3.23972	3.16987	3.10245	3.03735
5	4.32948	4.21236	4.10020	3.99271	3.88965	3.79079	3.69590	3.60478
6	5.07569	4.91732	4.76654	4.62288	4.48592	3.35526	4.23054	4.11141
7	5.78637	5.58238	5.38929	5.20637	5.03295	4.86842	4.71220	4.56376

Year	Beginning Principal 1	Annual Payment 2	Interest Expense 3 = 1 ×10%	Principal Reduction 4 = 2 - 3	Remaining Principal 5 = 1 - 4
1	97,368	20,000	9,736.8	10,263.2	87,104.8
2	87,104.8	20,000	8,710.48	11,289.52	75,815.28
3	75,815.28	20,000	7,581.53	12,418.47	63,396.81
4	63,396.81	20,000	6,339.68	13660.32	49,736.49
5	49,736.49	20,000	4,973.65	15,026.35	34,710.14
6	34,710.14	20,000	3,471.01	16,528.99	1,8181.15
7	18,181.15	20,000	1,818.85*	18,181.15	0
Total		140,000	42,632	97368	



Exercises:

1- Salman paid an annuity of BD250 at the end of each three months for 9 years at an interest rate of 4% quarterly. Find the following:

a- Future value (amount) and interest at the end of the period.

b- Present value (amount) at the end of the period.

2- A trader paid an annuity of BD900 at the beginning of each six months at an interest rate of 5% annually compounded semiannually.
Find the following:

a- Future value (amount) and interest at the end of 8 years.

b- Present value of the annuities at the end of the period

3- Calculate the future value and interest of an annuity due of BD800 paid 6 times a year for 5 years if the nominal rate is 3.5% annually.

4- Sajeda deposits a sum of money at the beginning of each year at 4% annually and the amount of annuity became BD1872.96 after 10 years. What was the value of each annuity?

5- Awatef paid at the end of each year sum of money at 2% annually so that the amount of annuities after 11 years was BD973.496. What was the value of each annuity?

6- Qassim deposits an equal annuity at the beginning of each year in his fund. The compound interest rate is 2.5% per annum and his balance at the end of 10 years was BD746.428. Find the value of each annuity.

7- Abdulla took a loan of BD20,000 from a bank by compound interest rate 12% annually for 8 years, compounded interest every 4 months.

8- Find the thirdly payment and prepare amortization loan schedule for first two years (first six payments).

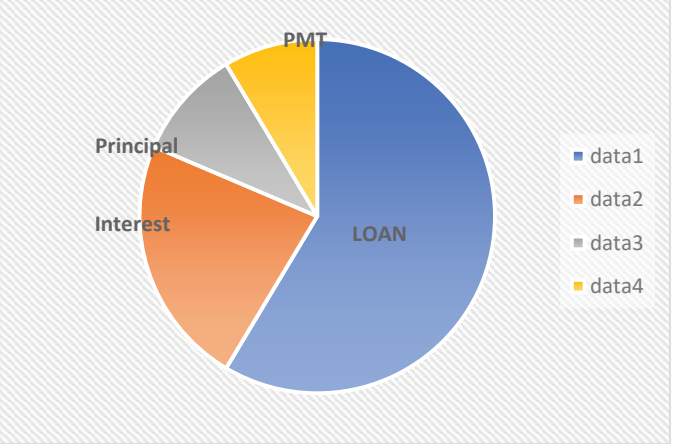
Exercises:

9- Find the annually payment and prepare amortization loan schedule for four years, if the loan is BD30,000 for 4 years at compound interest rate 5% annually.

10- You are required to prepare amortization loan schedule for a BD40,000 loan to be repaid in equal instalments at the end of each of the next three years. The interest rate is 9% annually, compounded interest semiannually.

11- Sara borrowed BD85,000 from a bank by compound interest rate 6% annually for 5 years. Find the annual payment and prepare amortization loan schedule for first five years.

12- Your family is planning to borrow BD110,000 to purchase a new house on 10-years,8% annual payment. What is the annual payment and prepare amortization loan schedule for the first four years?



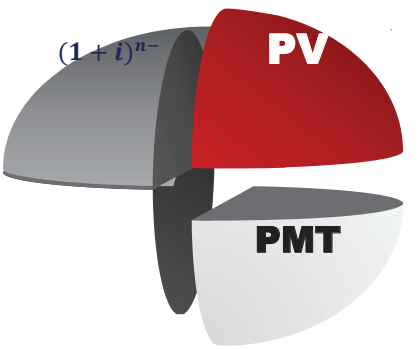


TABLE (PV of Ordinary Annuity) (annuity in arrears … end of period)									
n \ i	0.25%	0.50%	0.75%	1.00%	1.50%	2.00%	2.50%	3.00%	4.00%
1	0.99751	0.99502	0.99256	0.99010	0.98522	0.98039	0.97561	0.97087	0.96154
2	1.99252	1.98510	1.97772	1.97040	1.95588	1.94156	1.92742	1.91347	1.88609
3	2.98506	2.97025	2.95556	2.94099	2.91220	2.88383	2.85602	2.82861	2.77509
4	3.97512	3.95050	3.92611	3.90197	3.85438	3.80773	3.76197	3.71710	3.62990
5	4.96272	4.92587	4.88944	4.85343	4.78264	4.71346	4.64583	4.57971	4.45182
6	5.94785	5.89638	5.84560	5.79548	5.69719	5.60143	5.50813	5.41719	2.24214
7	6.93052	6.86207	6.79764	6.72819	6.59821	6.47199	6.34939	6.23028	6.00205
8	7.91074	7.82296	7.73661	7.65168	7.48593	7.32548	7.17014	7.01969	6.73274
9	8.88852	8.77906	8.67158	8.56602	8.36052	8.16224	7.97087	7.78611	7.43533
10	9.86386	9.73041	9.59958	9.47130	9.22218	8.98259	8.75206	8.53020	8.11090
11	10.83677	10.67703	10.52067	10.36763	10.07112	9.78685	9.51421	9.25262	8.76048
12	11.80725	11.61893	11.43491	11.25505	10.90751	10.57534	10.25776	9.95400	9.38507
13	12.27532	12.55615	12.34235	12.13374	11.73153	11.34837	10.98318	10.63496	9.98565
14	13.74096	13.48871	13.24302	13.00370	12.54338	12.10625	11.69091	11.29607	10.56312
15	14.66504	14.41662	14.13699	13.86505	13.34323	12.84926	12.38138	11.93794	11.11839
16	15.66504	15.33993	15.02431	14.71787	14.13126	13.57771	13.05500	12.56110	11.65230
17	16.62348	16.25863	15.90502	15.56225	14.90765	14.29187	13.71220	13.16612	12.16567
18	17.57953	17.17277	16.77918	16.39827	15.67256	14.99203	14.35336	13.75351	12.65930
19	18.53320	18.08236	17.64683	17.22601	16.42617	15.67846	14.97889	14.32380	13.13394
20	19.48449	18.98742	18.50802	18.04555	17.16864	16.35143	15.58916	14.87747	13.59033
21	20.43340	19.88798	19.36280	18.85698	17.90014	17.01121	16.18455	15.41502	14.02916
22	21.37995	20.78406	20.21121	19.66038	18.26802	17.65805	16.76541	15.93692	14.45112
23	22.32414	21.67568	21.05231	20.45582	19.33086	18.29220	17.33211	16.44361	14.85684
24	23.26598	22.56287	21.88915	21.24339	20.03041	18.91393	17.88499	16.93554	15.24696
25	24.20547	23.44564	22.71876	22.02316	20.71961	19.52346	18.42438	17.41315	15.62208
30	28.86787	27.79405	26.77508	25.80771	24.01584	22.39646	20.93029	19.60044	17.29203
35	33.47243	32.03537	30.68266	29.40858	27.07559	24.99862	23.14516	21.48722	18.66461
40	38.01986	36.17223	34.44694	32.83469	29.91585	27.35548	25.10278	23.11477	19.79277
50	46.94617	44.14279	41.56645	39.19612	34.99969	31.42361	28.36231	25.72976	21.48218

TABLE (PV of Ordinary Annuity) (annuity in arrears … end of period)								
n \ i	5.00%	6.00%	7.00%	8.00%	9.00%	10.00%	11.00%	12.00%
1	0.95238	0.94340	0.93458	0.92593	0.91743	0.90909	0.90090	0.89286
2	1.85941	1.83339	1.80802	1.78326	1.75911	1.73554	1.71252	1.69005
3	2.72325	2.67301	2.62432	2.57710	2.53129	2.48685	2.44371	2.40183
4	3.54595	3.46511	3.38721	3.31213	3.23972	3.16987	3.10245	3.03735
5	4.32948	4.21236	4.10020	3.99271	3.88965	3.79079	3.69590	3.60478
6	5.07569	4.91732	4.76654	4.62288	4.48592	3.35526	4.23054	4.11141
7	5.78637	5.58238	5.38929	5.20637	5.03295	4.86842	4.71220	4.56376
8	6.46321	6.20979	5.97130	5.74664	5.53482	5.33493	5.14612	4.96764
9	7.10782	6.80169	6.51523	6.24689	5.99525	5.75902	5.53705	5.32825
10	7.72183	7.36009	7.02358	6.71008	6.41766	6.14457	5.88923	5.65022
11	8.30641	7.88687	7.49867	7.13896	6.80519	6.49506	6.20652	5.93770
12	8.86325	8.38384	7.94269	7.53608	7.16073	6.81369	6.49236	6.19437
13	9.39357	8.85268	8.35765	7.90378	7.48690	7.10336	6.74987	6.42355
14	9.89864	9.29498	8.74547	8.24424	7.78615	7.36669	6.98187	6.62817
15	10.37966	9.71225	9.10791	8.55948	8.06069	7.60608	7.19087	6.81086
16	10.83777	10.10590	9.44665	8.85137	8.31256	7.82371	7.37916	6.97399
17	11.27407	10.47726	9.76322	9.12164	8.54363	8.02155	7.54879	7.11963
18	11.68959	10.82760	10.05091	9.37189	8.75563	8.20141	7.70162	7.24967
19	12.08532	11.15812	10.33560	9.60360	8.95011	8.36492	7.83929	7.36578
20	12.46211	11.46992	10.59401	9.81815	9.12855	8.51356	7.96333	7.46944
21	12.82115	11.76408	10.83553	10.01680	9.29224	8.64869	8.07507	7.56200
22	13.16300	12.04158	11.06124	10.20074	9.44243	8.77154	8.17574	7.64465
23	13.48857	12.30338	11.27219	10.37106	9.58021	8.88322	8.26643	7.71843
24	13.79864	12.55036	11.46933	10.52876	9.70661	8.89474	8.34814	7.78432
25	14.09394	12.78336	11.65358	10.67478	9.82258	9.07704	8.42174	7.84314
30	15.37245	13.76483	12.40904	11.25778	10.27365	9.42691	8.69379	8.05518
35	16.37419	14.49825	12.94767	11.65457	10.56682	9.64416	8.85524	8.17550
40	17.15909	15.04630	13.33171	11.92461	10.75736	9.77905	8.95105	8.24378
50	18.25593	15.76186	13.80075	12.23348	10.96168	9.91481	9.04165	8.30450

TABLE (FV of Ordinary Annuity) (annuity in arrears … end of period)								
n \ i	0.25%	0.50%	0.75%	1.00%	1.50%	2.00%	2.50%	3.00%
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	2.00250	2.00500	2.00750	2.01000	2.01500	2.02000	2.02500	2.03000
3	3.00751	3.01502	3.02256	3.03010	3.04522	3.06040	3.07563	3.09090
4	4.01503	4.03010	4.04523	4.06040	4.09090	4.12161	4.15252	4.18363
5	5.02506	5.05025	5.07556	5.10101	5.15227	5.20404	5.25633	5.30914
6	6.03763	6.07550	6.11363	6.15202	6.22955	6.30812	6.38774	6.46841
7	7.05272	7.10588	7.15948	7.21354	7.32299	7.43428	7.54743	7.66246
8	8.07035	8.14141	8.21318	8.28567	8.43284	8.58297	8.73612	8.89234
9	9.09053	9.18212	9.27478	9.36853	9.55933	9.75463	9.95452	10.15911
10	10.11325	10.22803	10.34434	10.46221	10.70272	10.94972	11.20338	11.46388
11	11.13854	11.27917	11.42192	11.56683	11.86326	12.16872	12.48347	12.80780
12	12.16638	12.33556	12.50759	12.68250	13.04121	13.41209	13.79555	14.19203
13	13.19680	13.39724	13.60139	13.80933	14.23683	14.68033	15.14044	15.61779
14	14.22979	14.46423	14.70340	14.94742	15.45038	15.97394	16.51895	17.08632
15	15.26537	15.53655	15.81368	16.09690	16.68214	17.29342	17.93193	18.59891
16	16.30353	16.61423	16.93228	17.25786	17.93237	18.63929	19.38022	20.15688
17	17.34429	17.69730	18.05927	18.43044	19.20136	20.01207	20.86473	21.76159
18	18.38765	18.78579	19.19472	19.61475	20.48938	21.41231	22.38635	23.41444
19	19.43362	19.87972	20.33868	20.81090	21.79672	22.84056	23.94601	25.11687
20	20.48220	20.97912	21.49122	22.01900	23.12367	24.29737	25.54466	26.87037
21	21.53341	22.08401	22.65240	23.23919	24.47052	25.78332	27.18327	28.67649
22	22.58724	23.19443	23.82230	24.47159	25.83758	27.29898	28.86286	30.53678
23	23.64371	24.31040	25.00096	25.71630	27.22514	28.84496	30.58443	32.45288
24	24.70282	25.43196	26.18847	26.97346	28.63352	30.42186	32.34904	34.42647
25	25.76457	26.55912	27.38488	28.24320	30.06302	32.03030	34.15776	36.45926
30	31.11331	32.28002	33.50290	34.78489	37.53868	40.56808	43.90270	47.57542
35	36.52924	38.14538	39.85381	41.66028	45.59209	49.99448	54.92821	60.46208
40	42.01320	44.15885	46.44648	48.88637	54.26789	60.40198	67.40255	75.40126
50	53.18868	56.64516	60.39426	64.46318	73.68283	84.57940	97.48435	112.7969

TABLE (FV of Ordinary Annuity) (annuity in arrears … end of period)									
n \ i	4.00%	5.00%	6.00%	7.00%	8.00%	9.00%	10.00%	11.00%	12.00%
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	2.04000	2.05000	2.06000	2.07000	2.08000	2.09000	2.10000	2.11000	2.12000
3	3.12160	3.15250	3.18360	3.21490	3.24640	3.27810	3.31000	3.34210	3.37440
4	4.24646	4.31013	4.37462	4.43994	4.50611	4.57313	4.64100	4.70973	4.77933
5	5.41632	5.52563	5.63709	5.75074	5.86660	5.98471	6.10510	6.22780	6.35285
6	6.63298	6.80191	6.97532	7.15329	7.33593	7.52333	7.71561	7.91286	8.11519
7	7.89829	8.14201	8.39384	8.65402	8.92280	9.20043	9.48717	9.78327	10.08901
8	9.21423	9.54911	9.89747	10.25980	10.63663	11.02847	11.43589	11.85943	12.29969
9	10.58280	11.02656	11.49132	11.97799	12.48756	13.02104	13.57948	14.16397	14.77566
10	12.00611	12.57789	13.18079	13.81645	14.48656	15.19293	15.93742	16.72201	17.54874
11	13.48635	14.20679	14.97164	15.78360	16.64549	17.56029	18.53117	19.56143	20.65458
12	15.02581	15.91713	16.86994	17.88845	18.97713	20.14072	21.38428	22.71319	24.13313
13	16.62684	17.71298	18.88214	20.14064	21.49530	22.95338	24.52271	26.21164	28.02911
14	18.29191	19.59863	21.01507	22.55049	24.21492	26.01919	27.97498	30.09492	32.39260
15	20.02359	21.57856	23.27597	25.12902	27.15211	29.36092	31.77248	34.40536	37.27971
16	21.82453	23.65749	25.67253	27.88805	30.32428	33.00340	35.94973	39.18995	42.75328
17	23.69751	25.84037	28.21288	30.84022	33.75023	36.97370	40.54470	44.50084	48.88367
18	25.64541	28.13238	30.90565	33.99903	37.45024	41.30134	45.59917	50.39594	55.74971
19	27.67123	30.53900	33.75999	37.37896	41.44626	46.01846	51.15909	56.93949	63.43968
20	29.77808	33.06595	36.78559	40.99549	45.76196	51.16012	57.27500	64.20283	72.05244
21	31.96920	35.71925	39.99273	44.86518	50.42292	56.76453	64.00250	72.26514	81.69874
22	34.24797	38.50521	43.39229	49.00574	55.45676	62.87334	71.40275	81.21431	92.50258
23	36.61789	41.43048	46.99583	53.43614	60.89330	69.53194	79.54302	91.14788	104.6029
24	39.08260	44.50200	50.81558	58.17667	66.76476	76.78981	88.49733	102.1742	118.1552
25	41.64591	47.72710	54.86451	63.24904	73.10594	84.70090	98.34706	114.4133	133.3339
30	56.08494	66.43885	79.05819	94.46079	113.2832	136.3075	164.4940	199.0209	241.3327
35	73.65222	90.32031	111.4348	138.2369	172.3168	215.7108	271.0244	341.5896	431.6635
40	95.02552	120.7998	154.7620	199.6351	259.0565	337.8824	442.5926	581.8261	767.0914
50	152.6671	209.3480	290.3359	406.5289	573.7702	815.0836	1163.909	1668.771	2400.018

TABLE (PV of Annuity Due) **(annuity in advance … beginning of period)**

n \ i	0.25%	0.50%	0.75%	1.00%	1.50%	2.00%	2.50%	3.00%	4.00%
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	1.99751	1.99502	1.99256	1.99010	1.98522	1.98039	1.97561	1.97087	1.96154
3	2.99252	2.98510	2.97772	2.97040	2.95588	2.94156	2.92742	2.91347	2.88609
4	3.98506	3.97025	3.95556	3.94099	3.912200	3.88388	3.85602	3.82861	3.77509
5	4.97512	4.95050	4.92611	4.90197	4.85438	4.80773	4.76197	4.71710	4.62990
6	5.96272	5.92587	5.88944	5.85343	5.78264	5.71346	5.64583	5.57971	5.45182
7	6.94785	6.89638	6.84560	6.79548	6.69719	6.60143	6.50813	6.41719	6.24214
8	7.93052	7.86207	7.79464	7.72819	7.59821	7.47199	7.34939	7.23028	7.00205
9	8.91074	8.82296	8.73661	8.65168	8.48593	8.32548	8.17014	8.01969	7.73274
10	9.88852	9.77906	9.67158	9.56602	9.36052	9.16224	8.97087	8.78611	8.43533
11	10.86386	10.73041	10.59958	10.47130	10.22218	9.98259	9.75206	9.53020	9.11090
12	11.83677	11.67703	11.52067	11.36763	11.07112	10.78685	10.51421	10.25262	9.76048
13	12.80725	12.61893	12.43491	12.25508	11.90751	11.5753	11.25776	10.95400	10.38507
14	13.77532	13.55615	13.34235	13.13374	12.73153	12.34837	11.98318	11.63496	10.98565
15	14.74096	14.48871	14.24302	14.00370	13.54338	13.10625	12.69091	12.29607	11.56312
16	15.70420	15.41662	15.13699	14.86505	14.34323	13.84926	13.38138	12.93794	12.11839
17	16.66504	16.33993	16.02431	15.71787	15.13126	14.57771	14.05500	13.56110	12.65230
18	17.62348	17.25863	16.90502	16.56225	15.90765	15.29187	14.71220	14.16612	13.16567
19	18.57953	18.17277	17.77918	17.39827	16.67256	15.99203	15.35336	14.75351	13.65930
20	19.53320	19.08236	18.64683	18.22601	17.42617	16.67846	15.97889	15.32380	14.13394
21	20.48449	19.98742	19.50802	19.04555	18.16864	17.35143	16.58916	15.87747	14.59033
22	21.43340	20.88798	20.36280	19.85698	18.90014	18.01121	17.18455	16.41502	15.02916
23	22.37995	21.78406	21.21121	20.66038	19.62082	18.65805	17.76541	16.93692	15.45112
24	23.32414	22.67568	22.05331	21.45582	20.33086	19.29220	18.33211	17.44361	15.85684
25	24.26598	23.56287	22.88915	22.24339	21.03041	19.91393	18.88499	17.93554	16.24696
30	28.94004	27.93302	26.97589	26.06579	24.37608	22.84438	21.45355	20.18845	17.98371
35	33.55611	32.19555	30.91278	29.70267	27.48173	25.49859	23.72379	22.13184	19.41120
40	38.11491	36.35309	34.70529	33.16303	30.36458	27.90259	25.73034	223.80822	20.58448
50	47.06354	44.36350	41.87820	39.58808	35.52468	32.05208	29.07137	26.50166	22.34147

TABLE (PV of Annuity Due) (annuity in advance … beginning of period)								
n \ i	5.00%	6.00%	7.00%	8.00%	9.00%	10.00%	11.00%	12.00%
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	1.96154	1.94340	1.93458	1.92593	1.91743	1.90909	1.90090	1.89286
3	2.88609	2.83339	2.80802	2.78326	2.75911	2.73554	2.71252	2.69005
4	3.77509	3.67301	3.62432	3.57710	3.53129	3.48685	3.44371	3.40183
5	4.62990	4.46511	4.38721	4.31213	4.23972	4.16987	4.10245	4.03735
6	5.45182	5.21236	5.10020	4.99271	4.88965	4.79079	4.69590	4.60478
7	6.24214	5.91732	5.76654	5.62288	5.48592	5.35526	5.23054	5.11141
8	7.00205	6.58238	6.38929	6.20637	6.03295	5.86842	5.71220	5.56376
9	7.73274	7.20979	6.97130	6.74664	6.53482	6.33493	6.14612	5.96764
10	8.43533	7.80169	7.51523	7.24689	6.99525	6.75902	6.53705	6.32825
11	9.11090	8.36009	8.02358	7.71008	7.41766	7.14457	6.88923	6.65022
12	9.76048	8.88687	8.49867	8.13896	7.80519	7.49506	7.20652	6.93770
13	10.38507	9.38384	8.94269	8.53608	8.16073	7.81369	7.49236	7.19437
14	10.98565	9.85268	9.35765	8.90378	8.48690	8.10336	7.74987	7.42355
15	11.56312	10.29498	9.74547	9.24424	8.78615	8.36669	7.98187	7.62817
16	12.11839	10.71225	10.10791	9.55948	9.06069	8.60608	8.19087	7.81086
17	12.65230	11.10590	10.44665	9.85137	9.31256	8.82371	8.37916	7.97399
18	13.16567	11.47726	10.76322	10.12164	9.54363	9.02155	8.54879	8.11963
19	13.65930	11.82760	11.05909	10.37189	9.75563	9.20141	8.70162	8.24967
20	14.13394	12.15812	11.33560	10.60360	9.95011	9.36492	8.83929	8.36578
21	14.59033	12.46992	11.59401	10.81815	10.12855	9.51356	8.96333	8.46944
22	15.02916	12.76408	11.83553	11.01680	10.29224	9.64869	9.07507	8.56200
23	15.45112	13.04158	12.06124	11.20074	10.44243	9.77154	9.17574	8.64465
24	15.85684	13.30338	12.27219	11.37106	10.58021	9.88322	9.26643	8.71843
25	16.24696	13.55036	12.46933	11.52876	10.70661	9.98474	9.34814	8.78432
30	17.98371	14.59072	13.27767	12.15841	11.19828	10.36961	9.65011	9.02181
35	19.41120	15.36814	13.85401	12.58693	11.51784	10.60857	9.82932	9.15656
40	20.58448	15.94907	14.26493	12.87858	11.72552	10.75696	9.93567	9.23303
50	22.34147	16.70757	14.76680	13.21216	11.94823	10.90630	10.03624	930104

TABLE (FV of Annuity Due) (annuity in advance … beginning of period)								
n \ i	0.25%	0.50%	0.75%	1.00%	1.50%	2.00%	2.50%	3.00%
1	1.00250	1.00500	1.00750	1.01000	1.01500	1.02000	1.02500	1.03000
2	2.00751	2.01502	2.02256	2.03010	2.04522	2.06040	2.07563	2.09090
3	3.01503	3.03010	3.04523	3.06040	3.09090	3.12161	3.15252	3.18363
4	4.02506	4.05025	4.07556	4.10101	4.15227	4.20404	4.25633	4.30914
5	5.03763	5.07550	5.11363	5.15202	5.22955	5.30812	5.38774	5.46841
6	6.05272	6.10588	6.15948	6.21354	6.32299	6.43428	6.54743	6.66246
7	7.07035	7.14141	7.21318	7.28567	7.43284	7.58297	7.73612	7.89234
8	8.09053	8.18212	8.27478	8.36253	8.55933	8.75463	8.95452	9.15911
9	9.11325	9.22803	9.34434	9.46221	9.70272	9.94972	10.20338	10.46388
10	10.13854	10.27917	10.42192	10.56683	10.56326	11.16872	11.48347	11.80780
11	11.16638	11.33556	11.50759	11.68250	12.04121	12.41209	12.79555	13.19203
12	12.19680	12.39724	12.60139	12.80933	13.23683	13.68033	14.14044	14.61779
13	13.22979	13.46423	13.70340	13.94742	14.45038	14.97394	15.51895	16.08632
14	14.26537	14.53655	14.81368	15.09690	15.68214	16.29342	16.93193	17.59891
15	15.30353	15.61423	15.93228	16.25786	16.93237	17.63929	18.38022	19.15688
16	16.34429	16.69730	17.05927	17.43044	18.20136	19.01207	19.86473	20.76159
17	17.38765	17.78579	18.19472	18.61475	19.48938	20.41231	21.38635	22.41444
18	18.43362	18.87972	19.33868	19.81090	20.79672	21.84056	22.94601	24.11687
19	19.48220	19.97912	20.49122	21.01900	22.12367	23.29737	24.54466	25.87037
20	20.53341	21.08401	21.65240	22.23919	23.47052	24.78332	26.18327	27.67649
21	21.58724	22.19443	22.82230	23.47159	24.83758	26.29898	27.86286	29.53678
22	22.64371	23.31040	24.00096	24.71630	26.22514	27.84496	29.58443	31.45288
23	23.70282	24.43196	25.18847	25.97346	27.63352	29.42186	31.34904	33.42647
24	24.76457	25.55912	26.38488	27.24320	29.06302	31.03030	33.15776	35.45926
25	25.82899	26.69191	27.59027	28.52563	30.51397	32.67091	35.01171	37.55304
30	31.19109	32.44142	33.75417	35.13274	38.10176	41.37944	45.00027	49.00268
35	36.62056	38.336610	40.15272	42.07688	46.27597	50.99437	56.30141	62.27594
40	42.11824	44.37964	46.79483	49.37524	55.08191	61.61002	69.08762	77.66330
50	53.32165	65.92839	60.84721	65.10781	74.78807	86.27099	99.92146	116.1808

TABLE (FV of Annuity Due) (annuity in advance … beginning of period)									
i / n	4.00%	5.00%	6.00%	7.00%	8.00%	9.00%	10.00%	%11.00	12.00%
1	1.04000	1.05000	1.06000	1.07000	1.08000	1.09000	1.10000	1.11000	1.12000
2	2.12160	2.15250	2.18360	2.21490	2.24640	2.27810	2.31000	2.34210	2.37440
3	3.24646	3.31013	3.37462	3.43994	3.50611	3.7313	3.64100	3.70973	3.77933
4	4.41632	4.52563	4.63709	4.75074	4.86660	4.98471	5.10510	5.22780	5.35285
5	5.63298	5.80191	5.97532	6.15329	6.33593	6.52333	6.71561	6.91286	7.11519
6	6.89829	7.14201	7.39384	7.65402	7.92282	8.20043	8.48717	8.78327	9.08901
7	8.21423	8.54911	8.89747	9.25980	9.63663	10.02847	10.43589	10.85943	11.29969
8	9.58280	10.02656	10.49132	10.97799	11.48456	12.02104	12.57948	13.16397	13.77566
9	11.00611	11.57789	12.18079	12.81645	13.48656	14.19293	14.93742	15.72201	16.54874
10	12.48635	13.20679	13.97164	14.78360	15.64549	16.56029	17.53117	18.56143	19.65458
11	14.02581	14.91713	15.86994	16.88845	17.97713	19.14072	20.38428	21.71319	23.13313
12	15.62684	16.71298	17.88214	19.14064	20.49530	21.95338	23.52271	25.21164	27.02911
13	17.29191	18.59863	20.01507	21.55049	23.21492	25.01919	26.97498	29.09492	31.39260
14	19.02359	20.57856	22.27597	24.12902	26.15211	28.36082	30.77248	33.40536	36.27971
15	20.82453	22.65749	24.67253	26.88805	29.32428	32.00340	34.94973	38.18995	41.75328
16	22.69751	24.84037	27.21288	29.84022	32.75023	35.97370	39.54470	43.50084	47.88367
17	24.64541	27.13238	29.90565	32.99903	36.45024	40.30134	44.59917	49.39594	54.74971
18	26.67123	29.53900	32.75999	36.37896	40.44626	45.01846	50.15909	55.93949	62.43968
19	28.77808	32.06595	35.78559	39.99549	44.76196	50.16012	56.27500	63.20283	71.05244
20	30.96920	34.71925	38.99273	43.86518	49.42292	55.76453	63.00250	71.26514	80.69874
21	33.24797	37.50521	42.39229	48.00574	54.45676	61.87334	70.40275	80.21431	91.50258
22	35.61789	40.43048	45.99583	52.43614	59.89330	68.53194	78.54302	90.14788	103.6029
23	38.08260	43.50200	49.81558	57.17667	65.76476	75.78981	87.49733	101.1742	117.1552
24	40.64591	46.72710	53.86451	62.24904	72.10594	83.70090	97.34706	113.4133	132.3339
25	43.31174	50.11345	58.15638	67.67647	78.94442	92.32398	108.1818	126.9988	149.3339
30	58.32834	69.76079	83.80168	101.0730	122.3459	148.5752	180.9434	220.9132	270.2926
35	76.59831	94.83632	118.1209	147.9135	186.1021	235.1247	298.1268	379.1644	483.4631
40	98.82654	126.8398	164.0477	213.6096	279.7810	368.2919	486.8518	645.8269	859.1424
50	158.7738	219.8154	307.7561	434.9860	619.6718	888.4411	1280.299	1852.336	2688.020

Unit 3

Capital Budgeting Decision Model

Learning Objective

At this unit, our students will learn:

- The difference between a short-term and long-term decision.
- The calculation of the payback period.
- The calculation of the net present value.
- The calculation of the profitability index.

3.1 Difference between a short and long-term decision:

Short-Term and Long-Term Decisions

What is the difference between a short-term decision and long-term decision?

We can separate short-term decision a long-term decision into three dimensions:

1. Length of effect
2. Cost
3. Degree of information gathering prior to the decision

The longer the effect and the higher the cost associated with a decision, the greater the time and degree allotted to gathering information on choices and the more sophisticated or complex the decision model.

Short-Term Decision:

A short-term decision, you will decide today what to eat for your next meal. This decision may involve a set of choice with varying costs. The choice affects you for only a short period, and the difference in cost of the different menu options is relatively small. In addition, you will face the next meal selection.

Then there are long-term decisions, recall for a moment your decision concerning which college to attend. This decision affects you for a number of years and carries with it significant financial costs. Moreover, this choice may well have been a once-in-a-lifetime decision. Although you can change schools after your initial choice, you cannot pick a different school to attend every few hours as you can with your menu choice at mealtime.

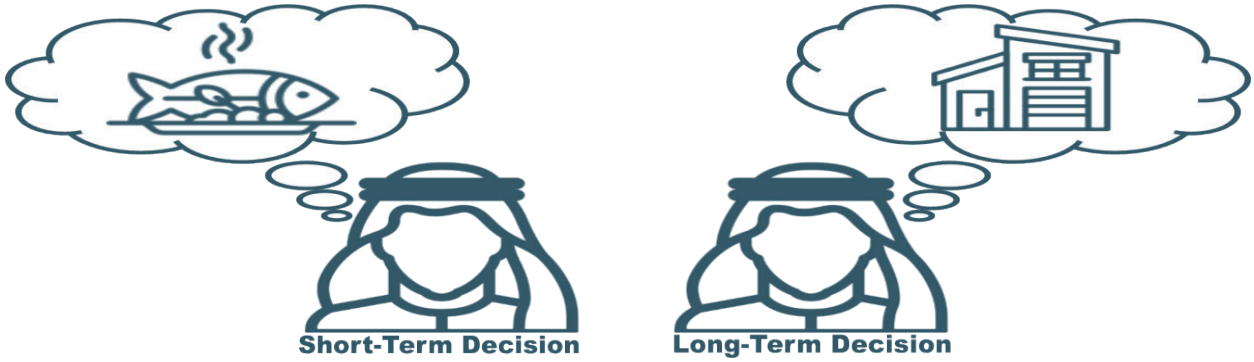


Figure (3-1-1): Type of Decision

Business use these dimensions when making choices about how to allocate money to products, services and activities of the firm. Long-term decisions are called **capital budgeting** decisions and are typically viewed as those that have long-term effects that are not easily reversed or that can be changed only at great cost.

Short-term decisions, on the other hand, are viewed as those that have short-term effects and can be changed or modified at relatively low costs.

By its nature, capital budgeting is concerned with long-term decision making and can be defined as the planning, appraising, comparing and selecting of a firm's long term projects. Long-term projects are those with lives that extend over a year, or longer than normal business operating cycle.

Tips 3-1-1:

We can make three key observations about capital budgeting decision:

1. A capital budgeting decision is typically a go or no-go decision on a product, service, facility or activity of the firm. That is, we either accept the business proposal or we reject it.
2. A capital budgeting decision will require sound estimates of the timing and amount of cash flow for the proposal.
3. The capital budgeting model has a predetermined accept or reject criterion. First, capital budgeting is about making decisions. The choice of accepting or rejecting a proposed project is the cornerstone of financial management at all levels of a business. Second, the appropriate future cash flow is a necessary input into all capital budgeting decisions.

3-1-1: Capital Budgeting Decision

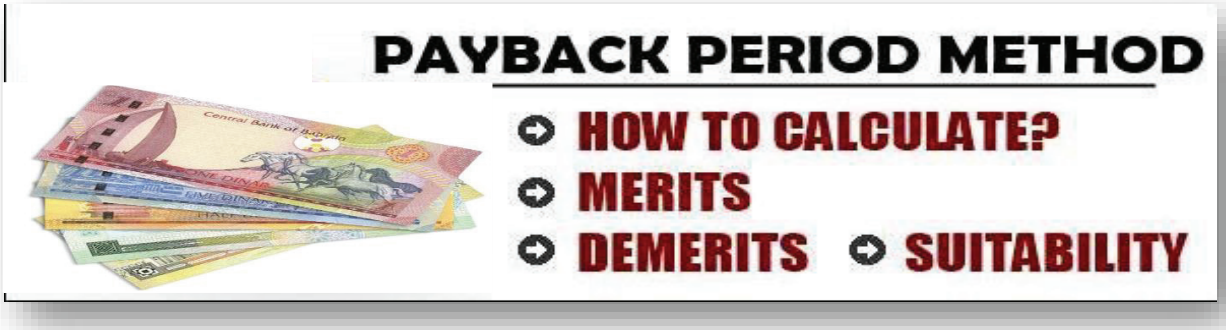
In this unit, we will introduce three standard models for capital budgeting decisions:

- Payback Period (standard)
- Net Present Value (NPV) (standard)
- Profitability Index (PI) (modified from the NPV)

3.2 Payback Period

1- Payback Period

Sofar the easiest decision model to administer is **payback period**. This model answers one basic question: How soon will I recover my initial investment? The model assumes there is an outflow of cash at the beginning of the project and a series of cash inflows during future periods. It simply calculates at what point in time the cash outflow is recovered or paid back the corresponding future cash inflow.



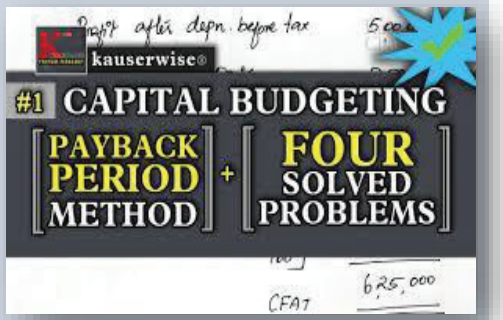
Example 3-2-1:

- Given the cash flow of three projects A, B and C, and using the payback period decision model, which projects do you accept and which project you will choose ?

Cash Flow	Project A	Project B	Project C
Cost (Cash Outflow)	BD10,000	BD10,000	BD10,000
Cash inflow year 1	BD3,500	BD2,000	BD5,500
Cash inflow year 2	BD3,500	BD3,400	BD4,500
Cash inflow year 3	BD3,500	BD4,000	BD4,000
Cash inflow year 4	BD3,500	BD5,000	BD3,000
Cash inflow year 5	BD3,500	BD6,000	BD2,000

Example 3-2-1:

Answer:



1. Project A: (Fixed Cash Inflow):

$$\textbf{Payback Period} = \frac{\textit{Cost (Initial Investment)}}{\textit{Annual Cash inflow}}$$

$$\text{Payback Period} = \frac{10000}{3500} = 2.86 \text{ years}$$

2. Project B: (Changeable Cash Inflow):

Year	Cash Flow BD	Yet to be recovered BD	Payback Period Year
0	-10,000		$= 3+\frac{600}{5000} = 3.12$ years
1	2,000	-10,000+2,000 = -8,000	
2	3,400	- 8,000+3,400 = -4,600	
3	4,000	- 4,600+4,000 = -600	
4	5,000	- 600+5,000 = 0 (recovered)	
5	6,000	Not used in decision	

3. Project C: (Changeable Cash Inflow):

Year	Cash Flow BD	Yet to be recovered BD	Payback Period Year
0	-10,000		2 years
1	5,500	-10,000 + 5,500 = - 4,500	
2	4,500	- 4,500 + 4,500 = 0 (recovered)	
3	4,000	Not used in decision	
4	3,000	Not used in decision	
5	2,000	Not used in decision	

Tips 3-2-1:

We can choose the project C because it has the lowest payback period. The company has required 2 years recover period is less than projects A and B. Although the payback period method is used widely, it has two significant weaknesses:

1. It ignored all cash flow after the initial cash outflow has been recovered.
2. It ignored the time value of money.

Advantages and Disadvantages of Payback Method:

Advantages of the Payback Method

The most significant advantage of the payback method is its simplicity. It's an easy way to compare several projects and then to take the project that has the shortest payback time. However, the payback has several practical and theoretical drawbacks.

Important Points 3-2-1:

Disadvantages of the Payback Method

1- **Ignores the time value of money**: The most serious disadvantage of the payback method is that it does not consider the time value of money. Cash flows received during the early years of a project get a higher weight than cash flows received in later years. Two projects could have the same payback period, but one project generates more cash flow in the early years, whereas the other project has higher cash flows in the later years. In this instance, the payback method does not provide a clear determination as to which project to select.

2- **Neglects cash flows received after payback period**: For some projects, the largest cash flows may not occur until after the payback period has ended. These projects could have higher returns on investment and may be preferable to projects that have shorter payback times.

Important Points 3-2-1:

Disadvantages of the Payback Method

3-Ignores a project's profitability: Just because a project has a short payback period does not mean that it is profitable. If the cash flows end at the payback period or are drastically reduced, a project might never return a profit and therefore, it would be an unwise investment.

4- **Does not consider a project's return on investment:** Some companies require capital investments to exceed a certain hurdle of rate of return; otherwise, the project is declined. The payback method does not consider a project's rate of return.

2- Net Present Value (NPV) (standard)

The capital budgeting decision model that uses all the discounted cash flows of a project is **net present value (NPV),** one of the single most important models in finance. The NPV of an investment is the present value of all benefits (cash in-flows) minus the present value of all costs (cash outflow) of the project. If the net amount is positive (benefits exceed costs), the project or choice is a "go" (accept).
If the net amount is negative (costs exceed benefits), the project or choice is a "no-go" (not-accept). If two projects are being compared, the one with the highest positive net present value is selected.

Tips 3-3-1:

$$NPV = -CF0 + \frac{CF1}{(1+r)^1} + \frac{CF1}{(1+r)^2} + \frac{CF1}{(1+r)^3} + \ldots + \frac{CFn}{(1+r)^n}$$

CF0 = Cash outflow (Costs = Initial investment)
CF1 = Cash inflow for the first year
CF2 = Cash inflow for the second year
CFn = Cash inflow for the last year
r = Discount rate

NPV

NET PRESENT VALUE

Accepting the project: If, NPV > 0

Rejecting the project: if, NPV < 0

Example 3-3-1:

- Given the cash flow of three projects A, B and C, and using the net present value decision model, which projects do you accept and which project you will choose?

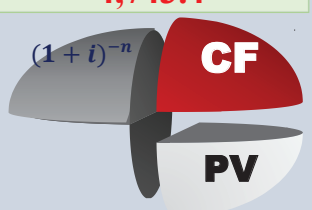
Cash Flow	Project A	Project B	Project C
Cost (Cash Outflow)	BD10,000	BD10,000	BD10,000
Cash inflow year 1	BD3,500	BD2,000	BD5,500
Cash inflow year 2	BD3,500	BD3,400	BD4,500
Cash inflow year 3	BD3,500	BD4,000	BD4,000
Cash inflow year 4	BD3,500	BD5,000	BD3,000
Cash inflow year 5	BD3,500	BD6,000	BD2,000
Discount rate	6%	6%	6%

Answer:

1. **Project A: (Fixed Cash Inflow):**

Year	Cash Flow (CF)	$\times (1+i)^{-n}$	PV of Cash flow
0	-10,000	$\times (1+6\%)^{0} = 1$	-10,000
1	3,500	$\times (1+6\%)^{-1} = 0.9434$	3,301.9
2	3,500	$\times (1+6\%)^{-2} = 0.8900$	3,115
3	3,500	$\times (1+6\%)^{-3} = 0.8396$	2,938.6
4	3,500	$\times (1+6\%)^{-4} = 0.7921$	2,772.35
5	3,500	$\times (1+6\%)^{-5} = 0.7473$	2,615.55
Net Present Value (NPV)			4,743.4

OR: Other Answer:



1- $PV_n = PMT \times PVIF\ n,r$

$= 3{,}500 \times 4.21236 \sim 4.42124 = BD14{,}473.4.$

2- NPV= 14,743.4 – 10,000 = BD4,743.4

PRESENT VALUE OF ORDINARY ANNUITY
(annuity in arrears -- end of period payments)

Periods	RATE PER PERIOD												
	0.25%	0.50%	0.75%	1.00%	1.50%	2.00%	2.50%	3.00%	4.00%	5.00%	6.00%	7.00%	8.00%
1	0.99751	0.99502	0.99256	0.99010	0.98522	0.98039	0.97561	0.97087	0.96154	0.95238	0.94340	0.93458	0.92593
2	1.99252	1.98510	1.97772	1.97040	1.95588	1.94156	1.92742	1.91347	1.88609	1.85941	1.83339	1.80802	1.78326
3	2.98506	2.97025	2.95556	2.94099	2.91220	2.88388	2.85602	2.82861	2.77509	2.72325	2.67301	2.62432	2.57710
4	3.97512	3.95050	3.92611	3.90197	3.85438	3.80773	3.76197	3.71710	3.62990	3.54595	3.46511	3.38721	3.31213
5	4.96272	4.92587	4.88944	4.85343	4.78264	4.71346	4.64583	4.57971	4.45182	4.32948	4.21236	4.10020	3.99271
6	5.94785	5.89638	5.84560	5.79548	5.69719	5.60143	5.50813	5.41719	5.24214	5.07569	4.91732	4.76654	4.62288

Example 3-3-1:

Answer:

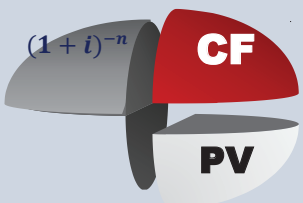
2. **Project B: (Changeable Cash Inflow):**

Year	Cash Flow (CF)	$\times (1+i)^{-n}$	PV of Cash flow
0	-10,000	$\times (1+6\%)^{0} = 1$	-10,000
1	2,000	$\times (1+6\%)^{-1} = 0.9434$	1,886.8
2	3,400	$\times (1+6\%)^{-2} = 0.8900$	3,026
3	4,000	$\times (1+6\%)^{-3} = 0.8396$	3,358.4
4	5,000	$\times (1+6\%)^{-4} = 0.7921$	3,960.5
5	6,000	$\times (1+6\%)^{-5} = 0.7473$	4,483.8
Net Present Value (NPV)			6,715.5

3. **Project C: (Changeable Cash Inflow):**

Year	Cash Flow (CF)	$\times (1+i)^{-n}$	PV of Cash flow
0	-10000	$\times (1+6\%)^{0} = 1$	-10,000
1	5,500	$\times (1+6\%)^{-1} = 0.9434$	5,188.7
2	4,500	$\times (1+6\%)^{-2} = 0.8900$	4,005
3	4,000	$\times (1+6\%)^{-3} = 0.8396$	3,358.4
4	3,000	$\times (1+6\%)^{-4} = 0.7921$	2,376.3
5	2,000	$\times (1+6\%)^{-5} = 0.7473$	1,494.6
Net Present Value (NPV)			6,423

- We accepted all three projects by using NPV method. The best we choose is project B because it has the highest NPV.



NPV

NET PRESENT VALUE

Important Points 3-3-1:

Advantages and Disadvantages of NPV Method:

❖ Advantages:

1- It considers the time value of money.
2- Helpful in decision-making.

❖ Disadvantages:

1- The calculation of the required rate of return does not have set guidelines.
2- Projects may be incomparable because of different investment terms and sizes.
3- Hidden costs are not considered.

3- Profitability Index (PI)

If people seem to have a natural affinity for rate of return rather than current Bahraini dinar for a project, is there a way to adjust the preferred NPV model and still have the ability to assign the appropriate level of risk for the project (the discount rate)? The answer again is a qualified yes, and the decision model to achieve this goal is the profitability index.

The **profitability index** (PI) is a modification of the NPV to produce the ratio of the present value of the benefits (future cash inflow) to the present value of the costs (initial investment):

Tips 3-4-1:

- The **profitability index** (PI) is a modification of the NPV to produce the ratio of the present value of the benefits (future cash inflow) to the present value of the costs (initial investment):

$$\text{Profitability Index} = \frac{Present\ Value\ of\ benefits}{Present\ Value\ of\ costs}$$

$$\text{Profitability Index} = \frac{NPV + Cost}{Cost}$$

$$\text{Profitability Index} = 1 + \frac{NPV}{Cost}$$

- The decision criterion is very straightforward: if PI > 1, accept the project; if PI < 1reject the project. Thus, when the PI is greater than 1, the benefits exceed the costs.
- If you already have the NPV of a project and it has a standard cash flow setup of all cash outflow at the beginning and cash inflow at a later period, you can quickly calculate the PI of the project.

Example 3-4-1:

- Using the results of example 3-2 to calculate profitability index of three projects.

Answer:

$$\text{PI project A} = \frac{4{,}743.4+10{,}000}{10{,}000} = 1.47434 > 1 \text{ (Accept)}$$

$$\text{PI project A} = \frac{6{,}715.5+1{,}0000}{10{,}000} = 1.67155 > 1 \text{ (Accept)}$$

$$\text{PI project A} = \frac{6{,}423+1{,}0000}{10{,}000} = 1.6423 > 1 \text{ (Accept)}$$

- We accepted all three projects by using PI method .The best we choose is project B because it has the highest PI.

Important Points 3-4-1:

Advantages and Disadvantages of PI Method:

❖ Advantages:

1. PI considers the time value of money.
2. PI considers analysis of all cash flows of entire life.
3. PI makes the right in the case of different amount of cash outlay of different project.
4. PI ascertains the exact rate of return of the project.

❖ Disadvantages:

1. It is difficult to understand the interest rate or discount rate.
2. It is difficult to calculate profitability index if two projects have different useful life.

Exercises:

Exercises (3-1):

The following three projects" their cash flow and using 8% discount rate.

Cash Flow	Project A BD	Project B BD	Project C BD
Cost (Initial Investment)	**20,000**	**20,000**	**20,000**
Cash flow year 1	6,000	8,000	3,000
Cash flow year 2	6,000	7,500	4,000
Cash flow year 3	6,000	6,000	5,000
Cash flow year 4	6,000	5,000	6,000

REQUIRED:

Which projects do you accept, and which projects do you reject by using:

a- Payback Period Method.
b- Net Present Value Method.
c- Profitability Index Method.

Exercises (3-2):

What are the payback periods projects R, S and T? Assume all the cash flow is evenly spread throughout the year. If the cutoff period is three years, which projects do you accept?

Cash Flow	Project R BD	Project S BD	Project T BD
Cost (Initial Investment)	**12,000**	**15,000**	**10,000**
Cash flow year 1	5,000	4,000	7,000
Cash flow year 2	8,000	6,500	3,500
Cash flow year 3	9,000	7,000	3,000

Exercises:

Exercises (3-3):

Mona Company has three projects, given discount rates and future cash flow of each project. Calculate the net present value of each project and identify which project do you accept?

Cash Flow	Project E BD	Project F BD	Project G BD
Cost (Initial Investment)	**14,000**	**18,000**	**22,000**
Cash flow year 1	3,200	9,100	5,700
Cash flow year 2	4,800	7,450	5,700
Cash flow year 3	5,300	7,000	6,250
Cash flow year 4	6,900	5,800	7,700
Discount rate	6%	7%	8%

Exercises (3-4):

Given the discount rates and future cash flow of each project listed. Use the profitability index method and identify which project do you accept?

Cash Flow	Project A BD	Project B BD	Project C BD
Cost (Initial Investment)	**16,000**	**16,000**	**16,000**
Cash flow year 1	5,000	6,200	3,650
Cash flow year 2	5,000	5,350	3,600
Cash flow year 3	5,000	5,150	4,550
Cash flow year 4	5,000	4,400	4,750
Discount rate	4%	5%	6%

Exercises:

Exercises (3-5):

Abdulla Company has three potential projects, all with an initial cost of BD 30,000. Given the discount rates and future cash flow of each project. Which project do you accept? By using:

a- Payback Period Method.
b- Net Present Value Method.
c- Profitability Index Method.

Cash Flow	Project R BD	Project S BD	Project T BD
Cash flow year 1	10,000	13,500	7,300
Cash flow year 2	10,000	12,000	8,800
Cash flow year 3	10,000	10,500	9,750
Cash flow year 4	10,000	7,200	11,600
Cash flow year 5	10,000	6,500	14,000
Discount rate	5.50%	5.50%	5.50%

Exercises (3-6):

ALAMAL Company has three potential projects, all with an initial cost of BD 75,000. Given the discount rates and future cash flow of each project. Which project do you accept? By using:

a- Payback Period Method.
b- Net Present Value Method.
c- Profitability Index Method.

Cash Flow	Project R BD	Project S BD	Project T BD
Cash flow year 1	30,000	15,500	25,000
Cash flow year 2	28,000	23,000	25,000
Cash flow year 3	24,000	30,500	25,000
Cash flow year 4	21,000	32,200	25,000
Discount rate	7.00%	7.50%	8.00%

Unit 4

Break-even Analysis

Learning Objective

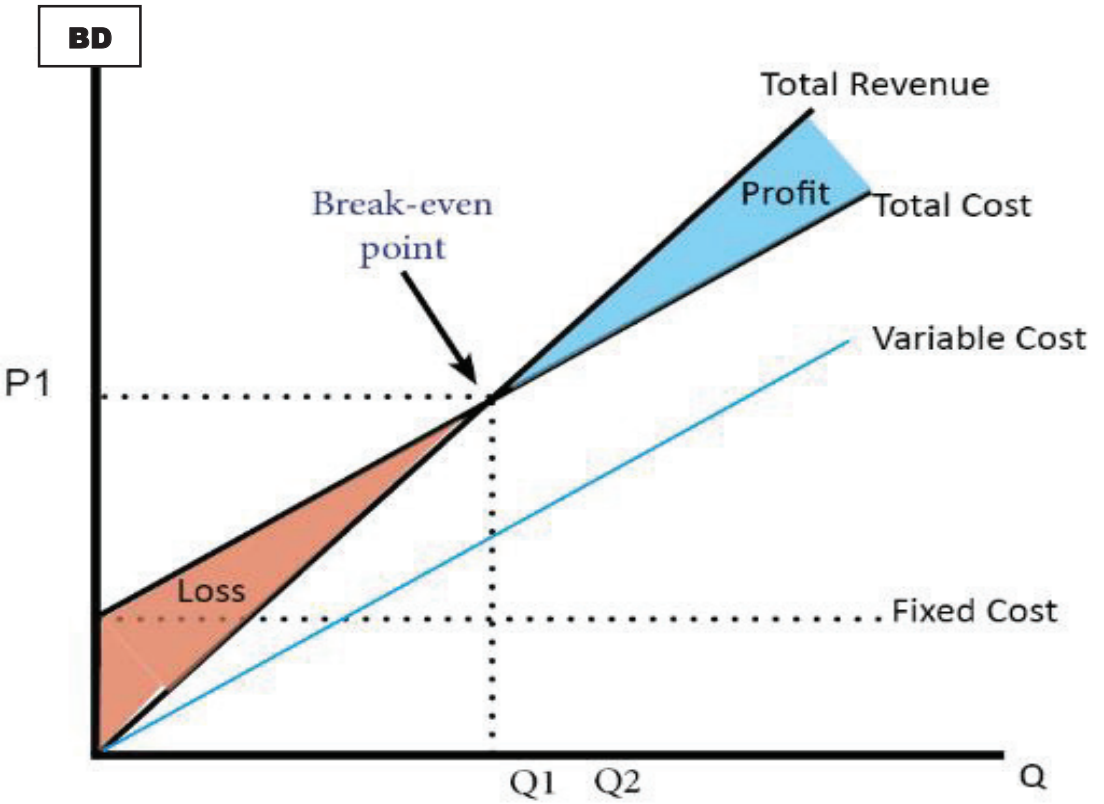
At this unit, our students will learn:

- The meaning of break-even point.
- The calculation of break-even point sales in units.
- The calculation of break-even point sales in Bahraini Dinar.
- The calculation of margin of safety in units.
- The calculation of sales in units to get target profit.
- The calculation of sales in Bahraini Dinar to get target profit.

4.1 Definition of Break-Even Point

The break-even point

In business, it is helpful to know how much output needs to be sold to cover costs. If costs are not covered by revenue, the business will make a loss. If revenue is greater than costs, the business will make a profit. If costs are exactly the same as revenue, the business will **break even.**



The chart is called a break - even chart because it shows the total revenues, costs and therefore total profit or loss of the business at the business at different levels of output. In the chart, the minimum level of output (Q1) that must be produced and sold each month is found at the point where its total revenue line crosses the total cost line. It shows that the business will break even if it produces and sell (Q1).

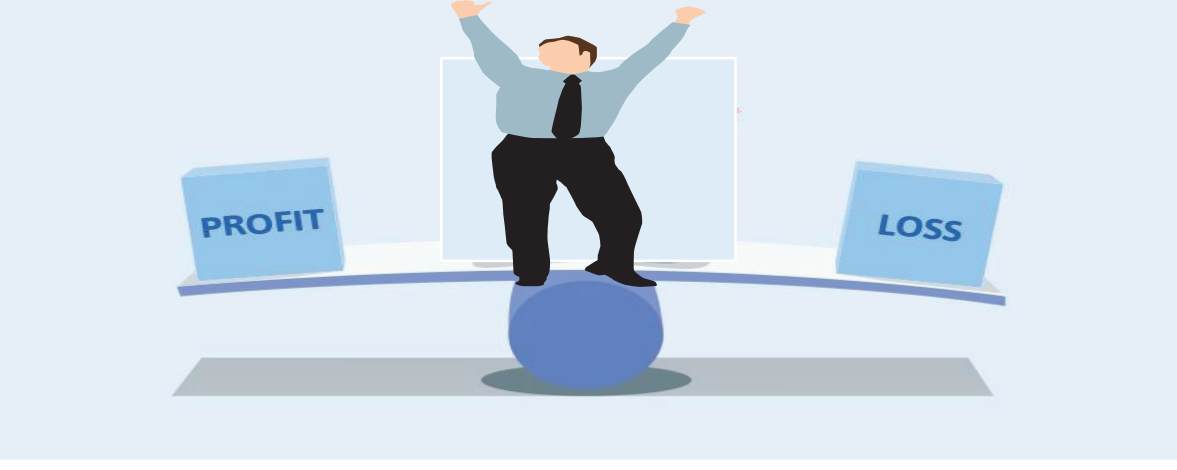
At every of output below (Q1), the business will make a loss. In the chart, the total revenue line will be below the total cost line. The area between them is the area of loss.
In contrast, at every level of output above (Q1) the business will make a profit. In the chart the total revenue line will be above total cost line and the area between them is therefore profit.

A business will break even if its total costs (TC) and total revenue (TR) are exactly the same. This is called the break-even point. At this point, the business does not make a profit or a loss.

Objective and uses of break-even

The objective of break-even analysis is to determine the break-even quantity of output by studying the relationships among the firm's cost structure, volume of output, and profit. Alternatively, the firm ascertains the break-even level of sales in Bahraini Dinar that corresponds to the break-even quantity of output. We will develop the fundamental relationships by concentrating on units of output, and then extend the procedure to permit direct calculation of the break-even sales level.

What is mean by the break-even quantity of output? It is the quantity of output, denominated in units, those results in an EBIT (earnings before interest and taxes) level equal to zero. Use of the break-even model, therefore, enables the financial officer (1) to determine the quantity of output that must be sold to cover all operating costs, as distinct from financial costs, and (2) to calculate the EBIT that will be achieved at various output levels.



Tips 4-1-1:

The many actual and potential applications of the break-even approach include the following:

1. **Capital expenditure analysis.** As a complementary technique to discounted cash flow evaluation models, the break-even model locates in a rough way the sales volume needed to make a project economically beneficial to the firm. It should not be used to replace the time-adjusted evaluation techniques.

2. **Pricing Policy.** The sales price of a new product can be set to achieve a target EBIT level. Furthermore, the market penetration should be a prime objective, and a price could be set that would cover slightly more than the variable costs of production and provide only a partial contribution to the recovery of fixed costs. The negative EBIT at several possible sales prices can then be studied.

3. **Labor contract negotiations.** The effect of increased variable costs resulting from higher wages on the break-even quantity of output can be analyzed.

4. **Cost structure.** The choice of reducing variable costs at the expense of incurring higher fixed costs can be evaluated. Management might decide to become more capital-intensive by forming tasks in the production process through use of equipment rather than labor. Application of the break-even model can indicate what the effects of this trade-off will be on the break-even point for the given product.

5. **Financing decisions.** Analysis of the firm's cost structure will reveal the proportion that fixed operation costs bear to sales. If this proportion is high, the firm might reasonably decide not to add any fixed financing costs on top of the high fixed operation costs.

4.2 Calculating the Break-Even point.

4-2-1: Calculate the break-even point:

To calculate the break-even point, the following information is needed:

- Fixed Cost.(FC)
- Variable Cost per Unit. (UVC)
- Selling Price per Unit. (USP)
- Unit Contribution Margin (UCM)

Important Points 4-2-1: Formulas of break-even point.

1- Unit Contribution Margin = Selling price per unit - Variable cost per unit

$$UCM = USP - UVC$$

2- Contribution Margin Percentage $= \frac{\text{Selling price per unit} - \text{Variable cost per unit}}{\text{Selling price per unit}} \times 100$

$$CM\% = \frac{USP-UVC}{USP} \times 100$$

3- Break-even point sales in units $= \frac{\text{Fixed Cost}}{\text{Selling price per unit} - \text{Variable cost per unit}}$

$$Unit\ Sales\ /_{BEP} = \frac{FC}{USP-UVC}$$

4- Break-even point sales in BD $= \frac{\text{Fixed Cost}}{\text{Contribution Margin Percentage}}$

$$Sales\ in\ BD\ /_{BEP} = \frac{FC}{CM\%}$$

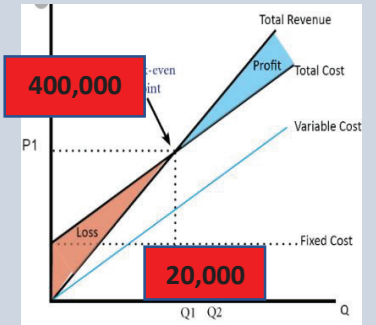
OR

Break-even point in sales BD = Break-even point in sales units × Unit Selling Price

$$= Unit\ Sales\ /_{BEP} \times USP\ Sales\ in\ BD\ /_{BEP}$$

Example 4-2-1:

ABC Company sold a computer at BD20 per unit and it had variable cost of BD12 per unit. The total annual fixed cost is BD160,000.



Required:

1) Calculate contribution margin per unit.
2) Calculate contribution margin percentage.
3) Calculate break-even point sales in units.
4) Calculate break-even point sales revenue in BD.

Answer:

1- Unit Contribution Margin = Selling price per unit - Variable cost per unit

$$UCM = USP - UVC$$
$$= 20 - 12 = BD8$$

2- Contribution Margin Percentage $= \frac{\text{Selling price per unit} - \text{Variable cost per unit}}{\text{Selling price per unit}} \times 100$

$$CM\% = \frac{USP-UVC}{USP} \times 100$$
$$= \frac{20-12}{20} \times 100 = 40\%$$

3- Break-even point sales in units $= \frac{\text{Fixed Cost}}{\text{Selling price per unit} - \text{Variable cost per unit}}$

$$Unit\ Sales\ /_{BEP} = \frac{FC}{USP-UVC}$$
$$= \frac{160{,}000}{20-12} = 20{,}000 \text{ Units}$$

4- Break-even point sales in BD $= \frac{\text{Fixed Cost}}{\text{Contribution Margin Percentage}}$

$$Sales\ in\ BD\ /_{BEP} = \frac{FC}{CM\%} =$$
$$\frac{160{,}000}{40\%} = BD400{,}000$$

OR

Break-even point in sales BD = Break-even point in sales units × Unit Selling Price

$$= Unit\ Sales\ /_{BEP} \times USP\ Sales\ in\ BD\ /_{BEP}$$
$$= 20{,}000 \times 20 = BD400{,}000$$

Example 4-2-2:

Refer to example 4-2-1: ABC company sold a computer at BD20 per unit and it had variable cost of BD12 per unit. The total annual fixed cost is BD160,000. If the variable cost per unit increased to BD16.

Required: New break-even after changing variable cost per unit

1) Calculate break-even point sales in units.
2) Calculate break-even point sales in BD.

Answer:

1- Break-even point sales in units $= \frac{\text{Fixed Cost}}{\text{Selling price per unit} - \text{Variable cost per unit}}$

$$Unit\ Sales\ /_{BEP} = \frac{FC}{USP - UVC}$$

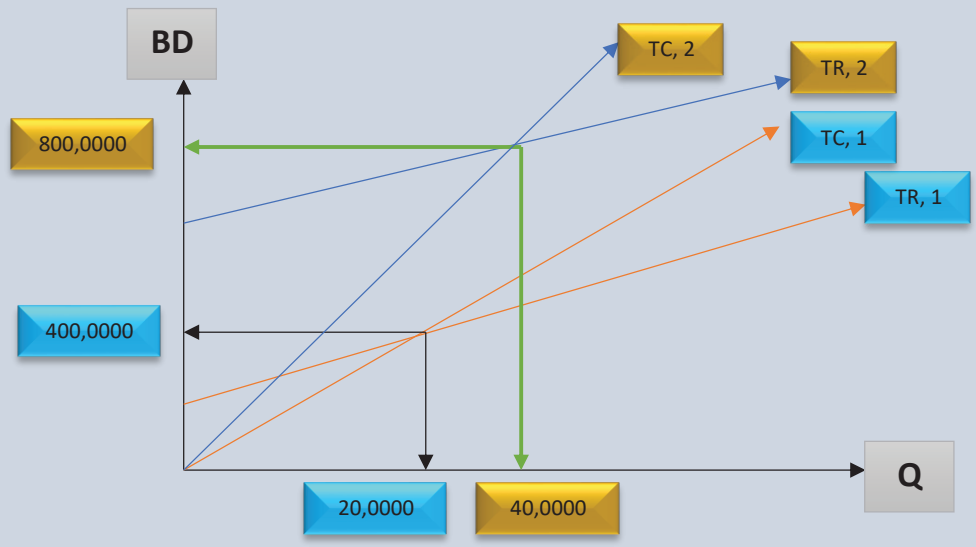
$$= \frac{160{,}000}{20-16} = 40{,}000\ \text{Units}$$

2-Break-even point in sales BD = Break-even point in sales units × Unit Selling Price

$$Sales\ in\ BD\ /_{BEP} = Unit\ Sales\ /_{BEP} \times USP$$

$$= 40{,}000 \times 20 = BD800{,}000$$

- If the company has an increase in variable cost per unit from BD12 to BD16, it must increase the output from 20,000 units to 40,000 units to get the break-even point.



Example 4-2-3:

► ABC Company sold a computer, it had variable cost BD12 per unit and the total annual fixed cost is BD160,000. If the break-even sales was 20,000 units.

Required:

Find the selling price per unit.

Answer:

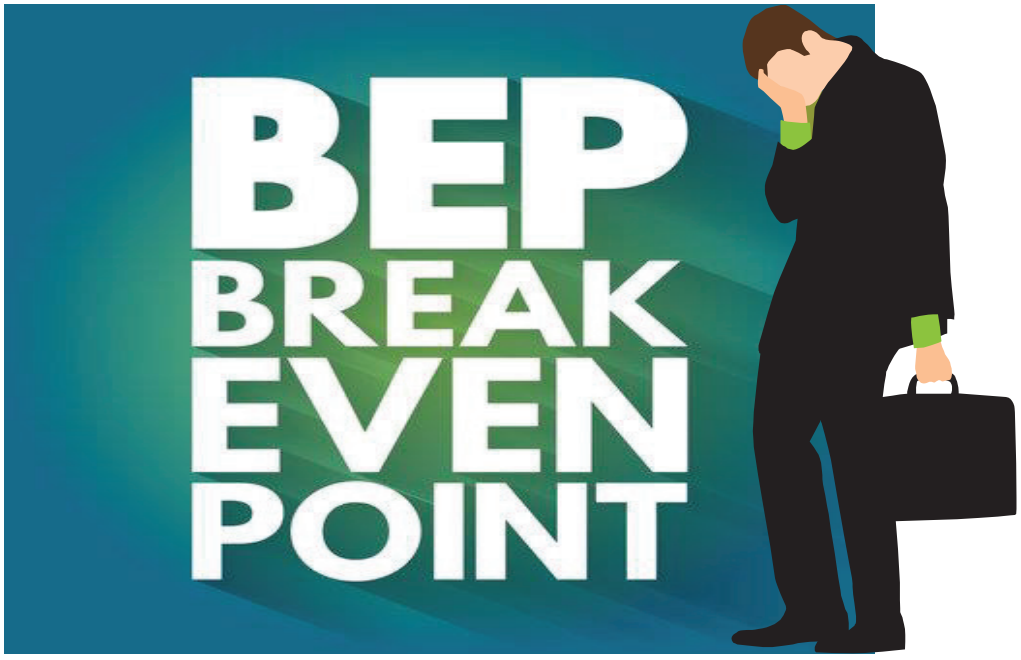
$$Unit\ Sales\ /_{BEP} = \frac{FC}{USP - UVC}$$

$$20{,}000 = \frac{160{,}000}{P - 12}$$

$$P - 12 = \frac{160{,}000}{20{,}000}$$

$$P - 12 = 8$$

$$P = 8 + 12 = BD20$$

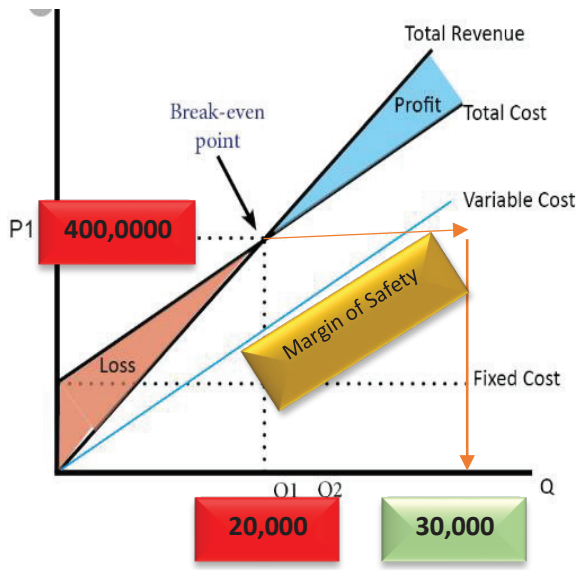


4.3 Margin of Safety and Target of Profit

Margin of Safety

A business should plan to have margin of safety by producing and selling more than its needs to break-even.

All businesses aim to make a profit by producing and selling products above the break-even output. ABC company has annual sales 30,000 units (actual sales) and break-even sales was 20,000 units. The margin of safety is the difference between the actual level of output and the break-even level of output which equals to 10,000 units. It can be calculated as follow:



Break-even analysis is used to make important production and pricing decisions

A break-even chart is a useful business-planning tool. Business owners and managers can use break-even charts to examine what might happen to their break-even level of output and profits costs or selling prices change.

For example, if costs rise, the amount of output a business must produce and sell each month or year to break-even also have to rise.

Important Points 4-3-1: Formula Margin of Safety.

Margin of Safety = Actual Unit Sales - Break-even point sales in units

= 30,000 - 20,000 = 10,000 Units

Target Operating Income (TOI)

Breaking even is not a bad thing, but hardly a satisfactory outcome for most businesses. Instead, a manager may be more interested in learning the necessary sales level to achieve a targeted profit. The approach to solving this problem is to treat the **target income** like an added increment of fixed costs. In other words, the margin must cover the fixed costs and the desired profit

Target Operating Income means the Operating Income projection for the measurement period agreed to by the Company's Board of Directors.

Example 4-3-1:

Refer to example (4-2-1): ABC Company sold a computer at BD20 per unit and it had variable cost BD12 per unit. The total annual fixed cost is BD160,000.

Required:

1- How many units must be sold to earn an operating income of BD40,000.
2- Compute sale revenue needed to earn an operating income of BD40,000.
3- If the company sold 30,000 units, what would be the company's profit or loss?

Answer:

1- Sales in units to earn income $= \frac{\text{Fixed Cost + Target Operating Income}}{\text{Selling price per unit} - \text{Variable cost per unit}}$

$$\text{Unit Sales}_{/\text{Target Income}} = \frac{FC+TOI}{USP-UVC}$$

$$= \frac{160{,}000 + 40{,}000}{20-12} = 25{,}000 \text{ Units}$$

2- Sales in revenue to earn income $= \frac{\text{Fixed Cost + Target Operating Income}}{\text{Contribution Margin Percentage}}$

$$\text{Sales in BD}_{/\text{Target Income}} = \frac{FC+TOI}{CM\%}$$

$$= \frac{160{,}000+40{,}000}{40\%} = \text{BD}500{,}000$$

OR

Sales in revenue to earn income = Sales in units to earn income × Unit Selling Price

$$\text{Sales in BD}_{/\text{Target Income}} = \text{Unit Sales}_{/\text{Target Income}} \times \text{USP}$$

$$= 25{,}000 \times 20 = \text{BD}500{,}000$$

Example 4-3-1:

Answer:

3- Profit/Loss = Units Sales × (Selling price per unit – Variable cost per unit) – Fixed Costs

Profit/Loss = [Q× (USP – UVC)] - FC =

= [30,000 × (20 – 12)] – (160,000)

= [30,000 × (8)] – (160,000)

= [240,000] – (160,000)

= BD80,000

Example 4-3-2:

- Abdulla Company expects to earn BD48,000 next year. Sales will be BD370,000, its average product sells for BD74 per unit. The variable cost per unit is BD50.

Required:

1- What are the company's fixed costs expected to be next year?

2- Calculate the company's break-even point in units and sales revenues?

Answer:

1- Unit Sales $= \frac{Sales}{\text{Selling price per unit}}$

$$Q = \frac{S}{USP} = \frac{370{,}000}{74} = 5{,}000 \text{ Units}$$

Profit/Loss = Units Sales × (Selling price per unit – Variable cost per unit) – Fixed Costs

Profit/Loss =[Q× (USP – UVC)] - FC =

48,000 = [5,000 × (74 – 50)] – (FC)

48,000 = [5,000 × (24)] – (FC)

FC = [120,000]– (48,000)

= BD72,000

Example 4-3-2:

Answer:

2- Break-even point sales in units $= \dfrac{\text{Fixed Cost}}{\text{Selling price per unit} - \text{Variable cost per unit}}$

$$Unit\ Sales\ /_{BEP} = \frac{FC}{USP - UVC}$$

$$= \frac{72{,}000}{74 - 50} = 3{,}000 \text{ Units}$$

2- Break-even point in sales revenue = Break-even point in sales units × Unit Selling Price

$$Sales\ in\ BD\ /_{BEP} = Unit\ Sales\ /_{BEP} \times USP =$$

$$= 3{,}000 \times 74 = BD222{,}000$$

The Limitations of break-even analysis

Break-even analysis is a useful decision tool in business but it has limitations. Business owners and manager need to be aware of these before making pricing and production decisions.

- Break-even charts assume the total output of a business will always be sold off, so there will be no unsold products left over. However, in reality most businesses build up stocks or inventories of their products in order to meet any unforeseen increase in consumer demand. This means a business may sometimes produce more than it sells. Similarly, a business may hold more materials and components than it needs just in case it has to increase production quickly to respond to a rise in consumer demand.
- Break-even analysis assumes fixed costs do not change. However, to expand output significantly, a business may need to invest in additional machinery, equipment and larger premises.
- Break-even analysis assumes selling prices are the same at all possible levels of output. In reality as a business increases its output, it may have to reduce its selling prices and therefore cut its profit margin, in order to persuade customers to buy the additional output.
- Market conditions are always changing. This affects the prices at which a business can sell its products.

- Break-even analysis needs accurate data on costs. A business producing a wide range of different goods or services may find it difficult to allocate different overheads and other costs to individual products, especially if their production shares the same premises, machinery, equipment and labour.
- The cost-volume-profit relationship is assumed to be linear. This is realistic only over narrow ranges of output.

Tips 4-3-1:

Key terms:

Break-even: The level of output where total costs and total revenue are exactly the same. Neither a profit nor a loss is made.

Break-even chart: A graph, which shows total cost and total revenue. The break-even point is where total cost and total revenue intersect.

Margin of safety: The amount of output available to be sold above the break-even point where the business make a profit.

Exercises:

Exercise (4-1):

AL Huda Company sell's office disk for BD30 each and estimated variable costs are expected to be 70% of sales. If the company's fixed costs are BD360,000.

Required:

1- How many office disk must the company sell to break-even?
2- Compute sales revenue for break-even.

Exercise (4-2):

Some financial data for each of three firms are as follow:

Title	Ahmed's Company	Osama's Company	Abdulla's Company
Selling price per unit	BD100	BD90	BD150
Variable cost per unit	BD80	BD60	BD120
Fixed Cost	BD140,000	BD120,000	BD180,000
Target Operating Income	BD60,000	BD30,000	BD90,000
Unit Sold	5,000 units	4,000 units	6,000 units

Required:

1- What is the break-even point in units and sales revenue for each company?
2- What is the units and sales revenue to get target profit?
3- What is the margin of safety in units?

Exercise (4-3):

Zeyad Corporation manufactures a line of computer, the average selling price of its finished product is BD180 per unit. The variable cost per unit is BD110. Zeyad incurs fixed costs of BD630,000.

Required:

1- What is the break-even point in units for the company?
2- What is the sales revenue the firm must achieve to reach the break-even point?

Exercises:

Exercise (4-4):

Footware Company manufactures a complete line of men's and women's dress shoes for independent merchants. The average selling price of its finished products is BD85 per pair. The variable cost for this same pair of shoes is BD58. Footware has fixed costs of BD270,000 per year.

Required:

1- What is the break-even point in " pair of shoes" for the company?
2- What is the sales revenue the firm must achieve to reach the break-even point?
3- If fixed cost increased to BD297,000. What is the new break-even point in pair of shoes and sales revenue?

Exercise (4-5):

Fahad Radios manufactures a complete line of radio and communication equipment. The average selling price of its finished products is BD180 per unit. The variable cost for these units is BD126. Fahad Radios incurs fixed costs of BD540,000 per year.

Required:

1- What is the break-even point in radio and communication equipment for the company?
2- What is the sales revenue the firm must achieve to reach the break-even point?
3- What would be the firm's profit or loss at the following units of production sold: 12,000 units? 15,000 units? 20,000 units?

Exercise (4-6):

Osama Company expects to earn BD40,000 next year. Sales will be BD400,000, its average product sells for BD20 per unit. The variable cost per unit is BD8.

Required:

1- What are the company's fixed costs expected to be next year?
1- Calculate the company's break-even point in units and sales revenues?

Exercises:

Exercise (4-7):

Hamad Company manufactures a full line of lawn furniture. The average selling price of its finished products is BD25 per unit. The associated variable cost for these units is BD15. Fixed costs of BD50,000 per year.

Required:

1- What is the break-even point in units for the company?
2- What is the sales revenue the firm must achieve to reach the break-even point?
3- What would be the firm's profit or loss at the following units of production sold: 4,000 units? 6,000 units? 8,000 units?
4- How many units must be sell to earn an operating income of BD30,000.
5- Compute sale revenue needed to earn an operating income of BD30,000.

Exercise (4-8):

Al Ahmed Company manufactures and sells pens. Currently, 5,000,000 units are sold per year at BD0.5 per unit. Fixed costs are BD900,000 per year. Variable costs are BD0.30 per unit.

Required: consider each case separately

1- a. What is the current annual profit or loss?
   b. What is the present break-even in units and revenues?
2- Compute the new profit or loss for each of the following changes:
   a. A BD0.04 per unit increase in variable costs.
   b. A 10% increase in fixed costs.
   c. A 205 decrease in selling price.

3- Compute the new break-even point in units and revenues for each of the following changes:
   a. A 20% increase in variable cost per unit.
   b. A BD20,000 increase in fixed costs.

Unit 5

Financial Ratio Analysis

Learning Objective

At this unit, our students will learn:

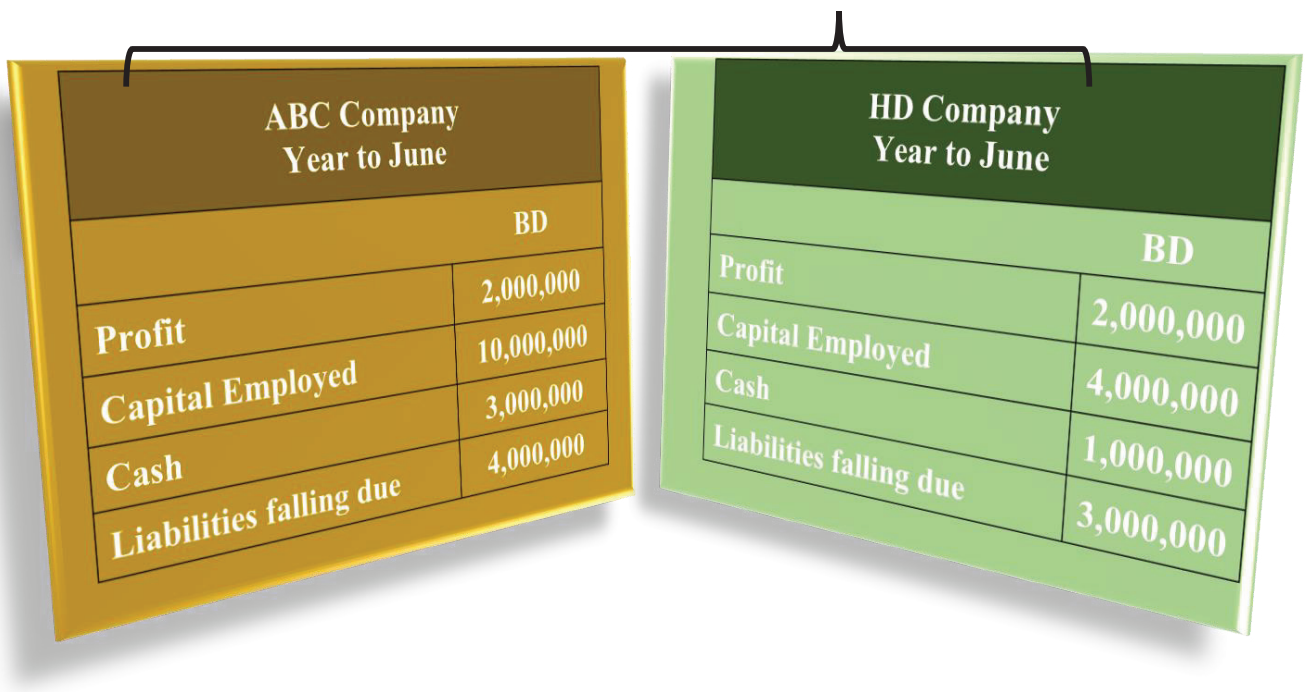
- The definition of the financial ratio.
- The calculation of and analyze profitability ratios.
- The benefits of liquidity ratio.
- The calculation of and analyze liquidity ratio.
- The benefits of liquidity ratio.

5.1 Interpreting Financial Performance

Ratio analysis involves comparing information in the financial statements of a business to measure the performance.

If a business is to survive and grow, it must make a profit and have enough cash to pay its bills. However, simply measuring how profit a business make each period and how much can it has doesn't tell us very much about financial strength or performance.

Most businesses aim to make a profit and calculate their profit as the difference between their revenues and costs each period using an income statement. However, some businesses are able to generate more profit from their resources than others. **For example**, compare the financial information on the two companies below.



ABC Company Year to June	BD
Profit	2,000,000
Capital Employed	10,000,000
Cash	3,000,000
Liabilities falling due	4,000,000

HD Company Year to June	BD
Profit	2,000,000
Capital Employed	4,000,000
Cash	1,000,000
Liabilities falling due	3,000,000

Both ABC Company and HD Company made the same amount of profit last year. However, ABC Company is a much larger business organization than HD Company.

The owners of ABC Company have invested BD10 million of capital in their business. A profit of BD2 million therefore represents a return of 20% on their investment. In contrast, the owner's capital invested in HD Company only BD4 million.

A profit of BD2 million therefore provides its owners with a 50% return on their investment. HD Company is therefore a more profitable business because it generates more profit from each BD1 also much better than to invest in ABC Company.

However, ABC Company held more cash than HD Company last year but it also had far bigger debts or liabilities to pay off than "smart buy". In fact, "better buy" had a liquidity problem because it didn't have enough cash or other assets it could convert to cash to pay off all its debts in need of settlement that year.

Simply looking at the amount of profit each business makes and how much cash it holds therefore tells us nothing about their **profitability** or **liquidity.** Analyzing financial information from business accounts by comparing two or more figures, such as profit as a percentage of capital employed or the ratio of cash to current liabilities provides a much better picture of business performance and is called **ratio analysis**.

Ratio Analysis

- **Ratio analysis** can be used to monitor the performance of a business over time and to compare the performance of different businesses.
- Cash flow forecasts, income statements and statements of financial position contain important information that can be examined by stakeholders interested in how well or bad different businesses are performing.

Tips 5-1-1:

An **accounting ratio** or **financial ratio** is simply the comparison of two figures in the financial statements of a business. A ratio is produced by dividing one key figure by another and in many cases taking a percentage. Accounting ratios are a good way to monitor how the performance of a business has changed over time and how it compares with other businesses. Many different accounting ratios can be calculated. The main ones used to analyse business performance are:

- **Profitability ratios**, which measure how well a business is using its assets to generate profits.
- **Liquidity ratios,** which measure the ability of a business to pay its short-term debts (or current liabilities) as they fall due from its cash and other liquid assets including its inventories and debtors.

5-2 : Profitability Ratios

Calculating and analyzing profitability ratios

Profit is a surplus of revenue over costs and is an absolute money amount. In contrast, **profitability** measures and compares profit relative to business size. In doing so, profitability measures how well or efficiently the business is using its resources to generate profit compared to other businesses.

Although a business may earn a profit it will not be considered profitable if

- Its profit is less than rival business of a similar size selling the same or similar products; and/or
- The owners of the business could earn more profit if they invested their capital in another business venture or even in a bank savings account.

Increasing profitability is therefore one of the most important tasks of business owners and managers. Increasing profitability will increase the value of the business and will also make the business more attractive to new investors.

5-2-1: Gross Profit Margin %

Tips 5-2-1:

- **Profitability ratios measure how much profit has been made as a percentage of revenue or capital employed**
- A number of accounting ratios are used to measure and compare the profitability of different businesses. They are:
  - Gross profit margin
  - Profit margin
  - Return on capital employed (ROGE)

Analysing the gross profit margin

The gross profit margin measures the proportion of money left over in a business from its sales revenuse after it has paid off the cost of the goods it has sold. A gross profit

margin that is lower than the industry average could indicate that the business is under-pricing its products or its costs og goods sold are too high. Investors will be attracted to firms who have a gross profit margin that is above the industry average.

Important Points 5-2-1:

- The gross profit margin is calculated as follows:

$$\textbf{Gross profit margin (\%)} = \frac{\textbf{Gross profit}}{\textbf{Revenue}} \times 100$$

Example 5-2-1:

- Below are financial statements for Fahad Ltd, a major hotel resort in a popular Caribbean holiday destination. The resort opened two years ago and business managers at Fahad Ltd are confident that performance has improved over time following a major expansion in year 2 that added more rooms, pools and other leisure facilities. This was funded through a new issue of shares, which also helped the resort pay off some of its short- and long-term liabilities.
- We will use the accounts of Fahad Ltd to calculate and compare a number of key performance and liquidity ratios.

Fahad Ltd Income statements	BD million	
	Year 1	Years 2
Revenue	**150**	**200**
less cost of sales	90	100
Gross profit	**60**	**100**
less Expenses	15	20
Profit before tax	**45**	**80**
less tax	9	16
Profit after tax	**36**	**64**
Of which		
distributed profit	30	50
retained profit	6	14

Fahad Ltd Statement of financial position	BD million			
	End year 1		End year 2	
Non-current assets		221		295
Cash	10		30	
Inventories	20		15	
Accounts Receivable	14		5	
Current assets	**44**		**50**	
less current liabilities	40		25	
Working capital		4		25
Total assets – current liabilities		**225**		**320**
Financed by				
Non-current liabilities	150		120	
Shareholders' funds	75		200	
Capital employed		**225**		**320**

Example 5-2-1:

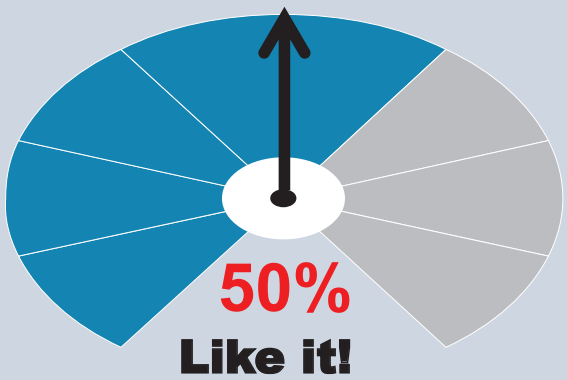
- Refer to the information in Example 5-2-1:

Required:

Find the gross profit margin % for year 1 and year2.

Answer:

	Year 1	Year 2
Gross profit (BD million) ÷ Revenue (BD million)	$\frac{60}{150} \times 100 = 40\%$	$\frac{100}{200} \times 100 = 50\%$



Using the information on gross profits and sales revenue from the income statements of Fahad Ltd on page ….. we can calculate and compare the gross profit margins over for first two accounting business years. These ratios show that:

- For every BD10 of sales in year 1 Fahad Ltd made a gross profit of BD4.
- In year 2 this had improved to BD5 gross profit from every BD10 revenue.
- The company has become more successful at generating a gross profit from its sales because it has been able to reduce the cost of sale and/ or rise the selling prices of its holidays.

Tips 5-2-1:

The gross profit margin will increase if:	The gross profit margin will decrease if:
• There is an increase in sale revenue. • The selling price of each item can be increased without a significant loss of sales, for example because consumer demand is rising and there are few other businesses to compete with. • Early payment discounts given to credit customers (debbtors) are reduced. • The cost of sale falls,for example because the business purchases items from cheaper suppliers or buys more in bulk to take advantages of discounts offered by suppliers.	• There is a fall in sales revenue • The selling price of each item is reduced, for exmple because competitors have reduced their prices. • The business offers more generous early payment discounts to credit customers in an attempt to boost credit sales and bring forward cash inflows. • The costs of sales rises, for example because suppliers have increased their selling prices.

5-2-2: Profit Margin %

The profit margin is another key indicator of the financial health of the business. It shows the amount of money left after costs and expenses have been paid. The profit margin tells investors and lenders how efficiently a business can convert the sale of its products to income.

Profit is calculated by deducting overheads from gross profit. The difference between gross profit and profit therefore gives an indication of a firm's ability to control its overhead costs. The more total revenues exceed total costs, the higher the profit margin.

The profit margin and gross profit margin provide a useful means of judging the performance of a business when comparing performance across two or more years. If gross margins stay constant but profit margins decrease, this means that overheads must have increased during the year. With this information, management may wish to improve cost control measures.

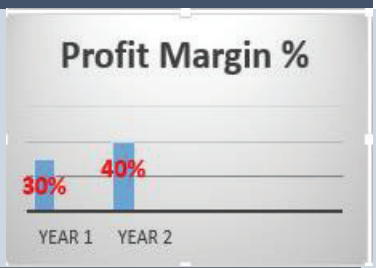
Important Points 5-2-2:

- The profit margin is calculated as follows:

$$\textbf{Profit margin (\%)} = \frac{\textbf{Profit before tax}}{\textbf{Revenue}} \times \textbf{100}$$

Example 5-2-2:

- Refer to the information in Example 5-2-1:



Required:

Find the profit margin % for year 1 and year2.

Answer:

	Year 1	Year 2
Profit before tax (million BD) ÷ Revenue (million BD)	$\frac{45}{150} \times 100 = 30\%$	$\frac{80}{200} \times 100 = 40\%$

These profit margins are a very good outcome for the company. They show the following.

- After deducting overheads from gross profit the company was still able to generate profit before tax of 3BD from every 10BD of revenue in first year.
- This improved in year 2 is 4BD of profit before tax from every 10BD of revenue.
- This was because gross profit had increased by 40 million BD compared to an increase in overhead costs of just 5 million BD.
- The company had become more successful at controlling its overheads or fixed costs for example because it was making more efficient use employees office and electricity and/or had found cheaper suppliers of cleaning and maintenance services.
- The profit margin for main competitor in years 1 and 2 was 30%.This means that Fahad Ltd has become more profitable than its rival

5-2-3: Return on Capital Employed % (ROCE)

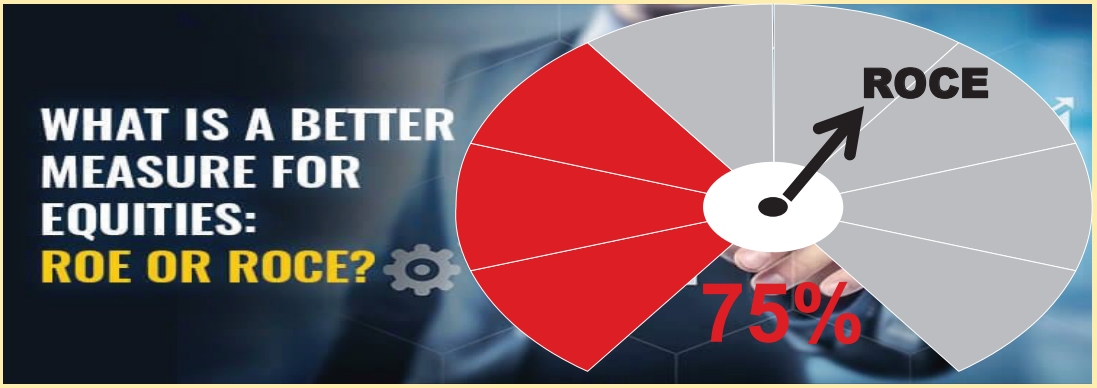
Analysing the return on capital employed (ROCE):

The ROCE expresses the profit of a company as a percentage of the total value of its capital employed. The ROCE of a business should ideally be higher than the rate of interest. Owners could earn by simply saving the same amount of mony in a bank account. This is because business owners are taking more risk with their money than savers. If the ROCE is lower than the return on savings, the owners would be better off selling their business assets for cash and putting this in a savings account earning inrerest.

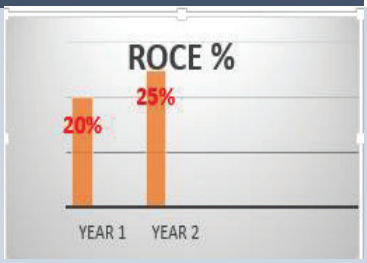
In limited companies the business owners are its shareholders and they expect to be paid a dividend from company profit. They will clearly want to earn more from their money invested in shares than they could get from interest on a savings account or from investing their money in another business venture. The ROCE ratio allows them to compare all these alternatives.

Important Points 5-2-3:

- The Return on capital employed is calculated as follows:
- $$\text{ROCE (\%)} = \frac{\text{Profit before tax}}{\text{Capital Employed}} \times 100$$
- **Where**
- **Capital Employed = Non-current liabilities + Shareholders Funds**
- **OR = Total assets – Current liabilities**



Example 5-2-3:



▶ Refer to the information in Example 5-2-1:

Required:

Find the ROCE % for year 1 and year2.

Answer:

	Year 1	Year 2
Profit before tax (million BD) ÷ Capital employed (million BD)	$\frac{45}{225} \times 100 = 20\%$	$\frac{80}{320} \times 100 = 25\%$

The higher ROCE of a company, better than for shareholders. This is because more profit, more dividends they will receive, or they can reinvest in their company to expand ability to generate additional revenues and profits in the future. The rate of return on the capital employed in Fahad Ltd is calculated from profit recorded income statement and total assets less current liabilities from statement of financial position.

ROCE recorded for Fahad Ltd gives the following information.

- The amount of profit "returned" from each BD10 of capital employed in the company was BD2 in year 1.
- By year 2 this had increased to BD2.5 of profit from every BD10 of capital employed in the company.
- These returns were much more than the owners could have earned from interest payments had they simply saved their capital in a bank savings account instead.
- At 25% ROCE of Fahad Ltd was also much higher than the 18% return earned by its main competitor over the same period, thereby making the resort a more attractive investment for its owners.
- The operating efficiency or producivity of the company had increased. Although capital employed had increased from BD 225 million to BD320 million it had become more successful at using assets to generate profit. For example, increasing in the scale of the company had allowed it to benefit from substantial economies of scale.

Activity 5-2-1:

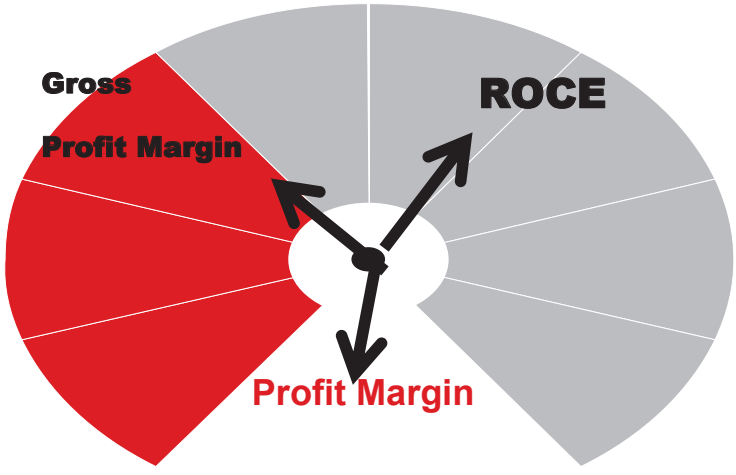
- MTA Wholesalesalers is a limited company operating in Bangladesh. It has been operating successfully for over 11 years. Below are some key results from its financial statements for the last two years.

MTA wholesalers Pvt Ltd Summarized results from financial statements		
	Year 10 BD (000)	**Year 11 BD (000)**
Revenue	400	420
Cost of sales	240	252
Gross profit	?	?
Overheads	130	147
Profit for the year	?	?
Capital employed	200	210

Required:

From the information presented calculate each year the company's:

- Gross Profit %.
- Gross Profit Margin %.
- ROCE%.



5-3: Liquidity Ratios

Liquidity measures the money available to pay business debts

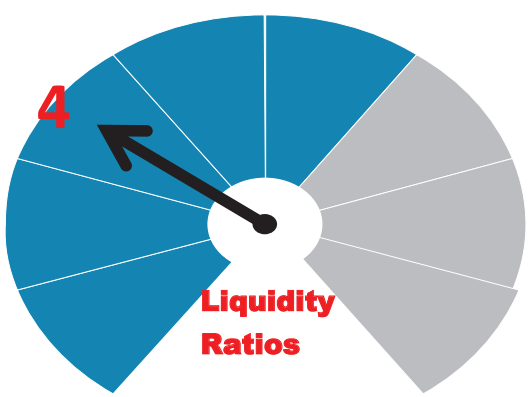
The liquidity of a business is measured by how quickly and easily it can raise cash to pay off its debts. A liquid asset is either cash or an asset that can quickly be converted into cash . debtors and stock will be sold quickly its examples for liquid assets.

A business will be in a good liquidiv position if it holds enough cash and other current assets can be easily convert to cash in order to meet current liabilities. A business is unable to pay debts on time is called **illiquid**. A business that is illiquid:-

- may have to obtain an expensive bank loan or sell off important assets, such as machinery, to raise cash. The loss of assets could reduce the amount of output the business could make and sell in future.

- may be forced to close down if it cannot raise cash. The business would be bankrupt.

In the short-term, liquidity is more important than profitability. However, all businesses need to make a profit so in the long term profitability is important. **Liquidity ratios**, also known as **solvency ratios**, are useful as they can give an early warning of financial problems that might occur if there is an unexpected bill to pay and sudden need for cash in a business.



5-3-1: Current Ratio

Analysing the current ratio

The current ratio measures is ability of a business to pay it immediate and short-term debts (current liabilities):

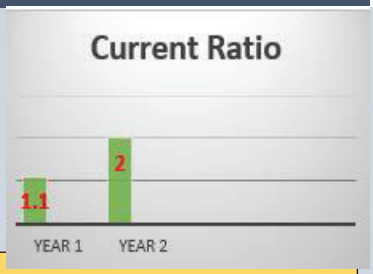
Important Points 5-3-1:

- The Current Ratio is calculated as follows:

$$\text{Current ratio} = \frac{\text{Current assets}}{\text{Current liabilities}}$$

Example 5-3-1:

- Refer to the information in Example 5-2-1:

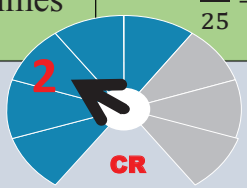


Required:

Find the current ratio for year 1 and year2.

Answer:

	Year 1	Year 2
Current assets (million BD) ÷ Current liabilities (million BD)	$\frac{44}{40}$ = 1.1 Times	$\frac{50}{25}$ = 2 Times



The following is shown by these results1.

- The current ratio at the end of year 1 was rather low. The company held BD1 in cash and other current assets for every BD1 it owed in current liabilities. Although the company could pay off its short-term debts from its current assets it had very little working capital **(current assets – current liabilities)** left over to meet any unexpected bills, for example a higher than anticipated electricity charge or equipment repair bill.

- By the end of year 2 the managers of Fahad Ltd had reduced the current liabilities to BD25 million and increased current assets to BD50 million. It did this by increasing cash sales and holding more cash in reserve. This means it will held now BD2 in current assets for every BD1 also owed in current liabilities.

Current Ratio

- Many businesses adopt a rule that current assets should be around double current liabilities to give a current ratio of two. Any lower, the business could be in dangerr of running out of cash and other current assets to meet its short-term debts and have enough working capital left over to continue financing its day-to-day running costs.
- A ratio of less than one means that the value of current liablities exceeds the value of current assets so there is negative working capital. A business will not be able to pay its immediate debts. This means it is insolvent. It may have to sell off some of its non-current assets to raise cash and may even be foreced to close.

Tips 5-3-1:

The current ratio will increase if:	The current ratio will decrease if:
1- the amount of cash of the business holds on its premises or the bank accounts is increased.	1- the amount of cash of the business holds on its premises or the bank accounts is reduced, for example due to falling cash sales.
2- current libailities are reduced, for example because the business pays off its creditors and bank overdraft using a long-term loan.	2- current liabilities increase, for example because the business increases its purchses on credit or overdraws its bank account.
3- overheads are reduced so cash outflows decrease.	3- overheads increase so cash outflows from the business increase to pay the higher costs.
4- purchases of equipment and other non-current assets are delayed so that the business holds on to its cash for longer.	4- cash spending on equipment and other non-current assets is increased.
5- the amount of profits paid out to the business owners is reduced (or dividends to shareholders are reduced) so the business retains more profit to hold as cash.	5- retained profits are reduced and paid to the business owners instead.

5-3-2: Quick Ratio

Analysing the acid test ratio (quick ratio).

The acid test ratio excludes inventories of finished products and materials from the valuation of current assets; the ratio measures whether or not a business is able to meet its short-term debts without having to sell off items it holds in stock.

The acid test ratio provides an alterntive ratio for measuring liquidity:

Important Points 5-3-2:

- The quick ratio is calculated as follows:

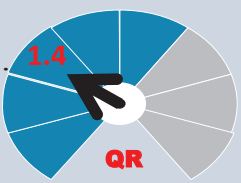
$$\text{Acid test ratio (Quick Ratio)} = \frac{(\text{Current assets} - \text{Inventories})}{\text{Current liabilities}}$$

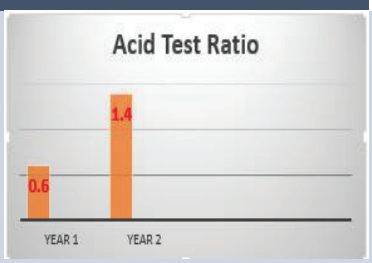
Example 5-3-2:

- Refer to the information in Example 5-2-1:

Required:

Find the quick ratio for year 1 and year2.





Answer:

	Year 1	Year 2
Current assets – Inventories (million BD) ÷ Current liabilities (million BD)	$\frac{44-20}{40}$ = 0.6 Times	$\frac{50-15}{25}$ = 1.4 Times

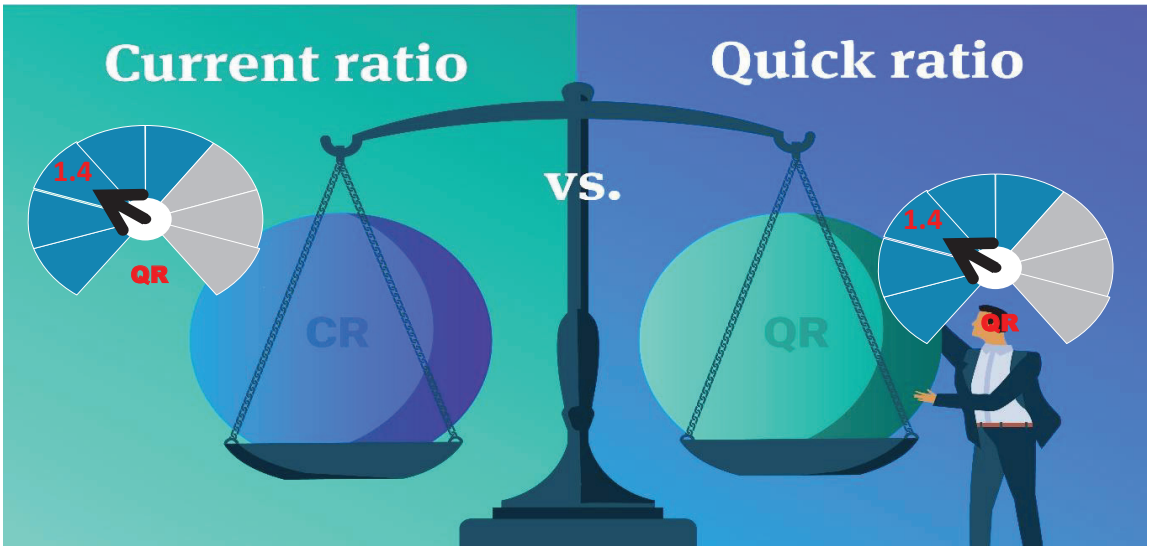
- Liquidity in the company at the end of year 1 was low. The company held only BD0.6 current assets other than invetories for every BD1 it owed in current liabilities. This mean it would be unable to pay off short-term debts in full without selling off inventories of food, drink, bed linen, towels and other items. The company would have become insolvent if it was unable to raise enough cash quickly from their sale.

- Recongnizing this danger, the managers of Fahad Ltd increased the cash holdings and reduced current liabilities. By the end of year 2 these actions had improved the acid test ratio of the company to BD1.4 for current assets other than inventories for every BD1 it owed in current liablilities.

Tips 5-3-2:

- In business, an acid test ratio of one – where the value of current assets less inventories is exactly equal to the value of current liabilities – is considered reasonably safe. This is because a business will be able to settle a short-term debts from cash and payments received from debtors without the need to sell off items holds in stock.

- If the ratio falls below a company owner's may be forced to sell off all inventories to pay off current liabilities. This may be difficult for example there is insufficient consumer demand for items.



Example 5-3-3:

- Below are some key resluts from financial statements prepared by the Arwa Company at the end of years 2019 and 2020.

Fahad Ltd Income statements	BD million	
	Year 2019	Years 2020
Revenue	500	900
less cost of sales	300	450
Gross profit	**200**	**450**
less expenses	40	225
Profit before tax	**160**	**225**
less tax	60	80
Profit after tax	**100**	**145**
Of which		
distributed profit	70	100
retained profit	30	20

Fahad Ltd Statement of financial position	BD million	
	End year 2019	End year 2020
Non-current assets	950	980
Cash	35	25
Inventories	14	20
Accounts Receivable	31	25
Current assets	**80**	**70**
less current liabilities	30	40
Working capital	50	30
Total assets – current liabilities	**1000**	**1010**
Financed by		
Non-current liabilities	200	200
Shareholders' funds	800	810
Capital Employed	**1000**	**1010**

Required:

From the information presented calculate for the company's each year its:

1- Gross profit margin %
2- Profit margin %
3- Return on Capital Employed . ROCE (%)
4- Current ratio.
5- Acid test ratio (Quick Ratio).

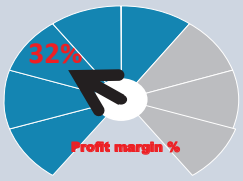
Answer:

1- Gross profit margin (%) = $\frac{\text{Gross profit}}{\text{Revenue}} \times 100$

	Year 2019	Year 2020
Gross profit (BD million) / Revenue (BD million)	$\frac{200}{500} \times 100 = 40\%$	$\frac{450}{900} \times 100 = 50\%$

Example 5-3-3:

Answer:

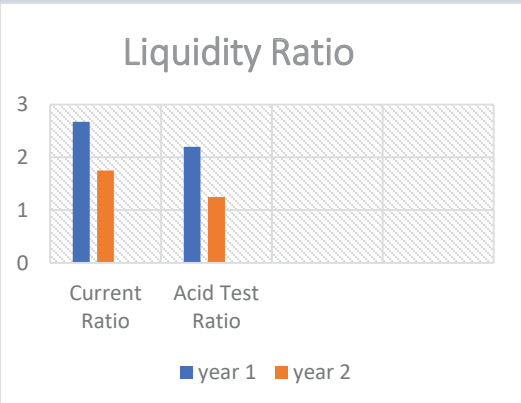


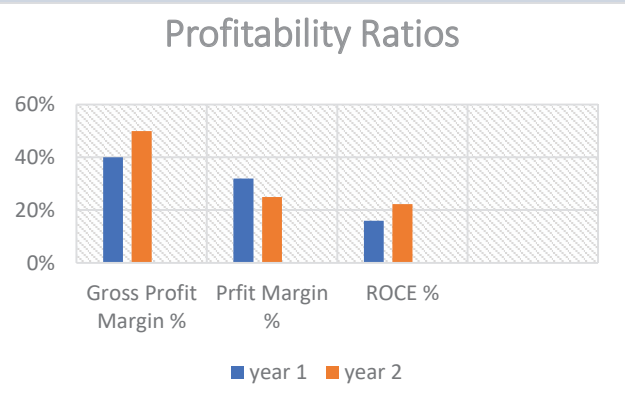
2- Profit margin (%) = $\frac{\text{Profit before tax}}{\text{Revenue}} \times 100$

	Year 2019	Year 2020
Profit before tax (BD million) Revenue (BD million)	$\frac{160}{500} \times 100 = 32\%$	$\frac{225}{900} \times 100 = 25\%$

3- ROCE (%) = $\frac{\text{Profit before tax}}{\text{Capital Employed}} \times 100$

	Year 2019	Year 2020
Profit before tax (million BD) Capital employed (million BD)	$\frac{160}{1000} \times 100 = 16\%$	$\frac{225}{1010} \times 100 = 22.28\%$





4- Current ratio = $\frac{\text{Current assets}}{\text{Current liabilities}}$

	Year 2019	Year 2020
Current assets (million BD) Current liabilities (million BD)	$\frac{80}{30} = 2.67$ Times	$\frac{70}{40} = 1.75$ Times

5- Acid test ratio (Quick Ratio) = $\frac{(\text{Current assets} - \text{Inventories})}{\text{Current liabilities}}$

(million BD)	Year 2019	Year 2020
Current assets – Inventories Current liabilities	$\frac{80-14}{30} = 2.2$ Times	$\frac{70-20}{40} = 1.25$ Times

Exercises:

Exercise (5-1):

The following results were extracted from statements of financial position prepared for Abdulla's Company at the end of years 2018 and 2019.

Abdulla's Company Summarized results from financial statements at 31 December		
	Year 2018 BD (000)	**Year 2019 BD (000)**
Cash	50	80
Inventories	40	60
Account Receivable	30	20
Total Current Assets	**120**	**160**
Account payable	15	20
Bank Overdraft	10	30
Total Current Liabilities	**25**	**50**

Required:

From the information presented calculate for the company's each year its:

1- Working Capital.
2- Capital Employed.
3- Current Ratio.
4- Acid Test Ratio (Quick Ratio)

Exercise (5-2):

The following as the income statements for Salman Company for years 2019 and 2020.

Required:

From the information presented, calculate for the company's each year its:

1- Gross Profit Margin %.
2- Profit Margin %.
3- Return on Capital Employed (%).

Salman Company Income Statements	BD million Year 2019	 Year 2020
Revenue	500	900
less cost of sales	300	450
Gross profit	**200**	**450**
less expenses	40	225
Profit before tax	**160**	**225**
less tax	60	80
Profit after tax	**100**	**145**

Exercises:

Exercise (5-3):

The balance sheet and income statement for MRG Company are as follows.

Balance Sheet	BD 000
Cash	500
Account Receivable	2,000
Inventories	1,000
Current Assets	**3,500**
Fixed Assets	4,500
Total Assets	**8,000**
Current Liabilities	2,000
Long-term debt	2,000
Owners' Equity	4,000
Total Liabilities and equity	**8,000**

Income Statement	BD 000
Net Sales (Revenues)	8,000
- Cost of Goods Sold	3,200
Gross Profit	**4,800**
- Operating Expenses	800
Operating Income	**4,000**
Interest Expenses	1,000
Profits before taxes	**3,000**
Tax (5%)	150
Net Income	**2,850**

Required:

Calculate the following ratios:

1- Gross Profit Margin %.
2- Profit Margin %.
3- Return on Capital Employed (%).
4- Working Capital.
5- Capital Employed.
6- Current Ratio.
7- Acid Test Ratio (Quick Ratio).

Exercise (5-4):

The annual sales for Sara Company were BD4.5 million last year, and the value of inventories was BD 120,000. The firm's end-of-year balance sheet and income statement appeared as follows:

Balance Sheet	BD 000
Current Assets	500,000
Net Fixed assets	1,500,000
Total assets	2,000,000
Current liabilities	400,000
Owner's Equity	1,600,000
Total liabilities and Equity	2,000,000

Income Statement	BD
Sales (Revenues)	4,500,000
- Cost of Goods Sold	3,500,000
Gross Profit	**1,000,000**
- Operating Expenses	(500,000)
Operating Income	**500,000**
Interest Expenses	100,000
Profits before taxes	**400,000**
Tax (5%)	20,000
Net Income	**380,000**

Exercises:

Required:

Calculate the following ratios:

1- Gross Profit Margin %.
2- Profit Margin %.
3- Return on Capital Employed (%).
4- Working Capital.
5- Capital Employed.
6- Current Ratio.
7- Acid Test Ratio (Quick Ratio).

Exercise (5-5):

The balance sheet and income statement for Sami's Company are as follows.

Balance Sheet	BD 000
Cash	1,000
Account Receivable	1,500
Inventories	1,000
Current Assets	**3,500**
Fixed Assets	4,500
Total Assets	**8,000**
Account Payable	1,000
Accrued Expenses	800
Total Current Liabilities	**1,800**
Long-term debt	2,100
Owners' Equity	4,100
Total Liabilities and equity	**8,000**

Income Statement	BD 000
Net Sales (Revenues)	8,000
- Cost of Goods Sold	3,200
Gross Profit	**4,800**
- Operating Expenses	800
Operating Income	**4,000**
Interest Expenses	1,000
Profits before taxes	**3,000**
Tax (5%)	150
Net Income	**2,850**

Required:

Calculate the following ratios:

1- Gross Profit Margin %.
2- Profit Margin %.
3- Return on Capital Employed (%).
4- Working Capital.
5- Capital Employed.
6- Current Ratio.
7- Acid Test Ratio (Quick Ratio).

Exercises:

Exercise (5-6):

Income Statement	2017 BD000	2018 BD000	2019 BD000	2020 BD000
Sales (Revenues)	3479	3644	3225	2,900
- Cost of Goods Sold	2109	2255	1997	1,746
Gross Profit	**1370**	**1389**	**1228**	**1,154**
- Operating Expenses	1105	1113	1088	1,015
Operating Income	**265**	**276**	**140**	**139**
Interest Expenses	35	56	20	29
Profits before taxes	**230**	**220**	**120**	**110**
Tax (5%)	11.5	11	6	5.5
Net Income	**218.5**	**209**	**114**	**104.5**

Balance Sheet	2017 BD000	2018 BD000	2019 BD000	2020 BD000
Cash	201	327	339	309
Account Receivable	507	591	562	518
Inventories	635	545	564	315
Current Assets	**1343**	**1463**	**1465**	**1362**
Fixed Assets	313	323	291	378
Total Assets	**1656**	**1786**	**1756**	**1740**
Current Liabilities	432	517	557	612
Long-term debt	254	888	692	587
Owners' Equity	970	381	507	541
Total Liabilities and equity	**1656**	**1786**	**1756**	**1740**

Required:

Calculate the following ratios for each year:

1- Gross Profit Margin %.
2- Profit Margin %.
3- Return on Capital Employed (%).
4- Working Capital.
5- Capital Employed.
6- Current Ratio.
7- Acid Test Ratio (Quick Ratio).

مطبعة أوال
awalpress

التعليم
مستقبل البحرين